# Advanced signal processing using NMF/NTF

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# Outline

1. Nonnegative matrix factorization (NMF)
2. NMF applications
3. Nonnegative tensor factorization (NTF)
4. NMF/NTF in condition monitoring
5. Orthogonal NMF as IFB selector

# Nonnegative matrix factorization

The input matrix $\boldsymbol{X} \in \mathbb{R}_+^{M \times N}$ can be approximated by two nonnegative matrices $\boldsymbol{W} \in \mathbb{R}_+^{M \times R}$ and $\boldsymbol{H} \in \mathbb{R}_+^{R \times N}$ according to the following equation:

$$\boldsymbol{X} \approx \boldsymbol{W}\boldsymbol{H}, \tag{1}$$

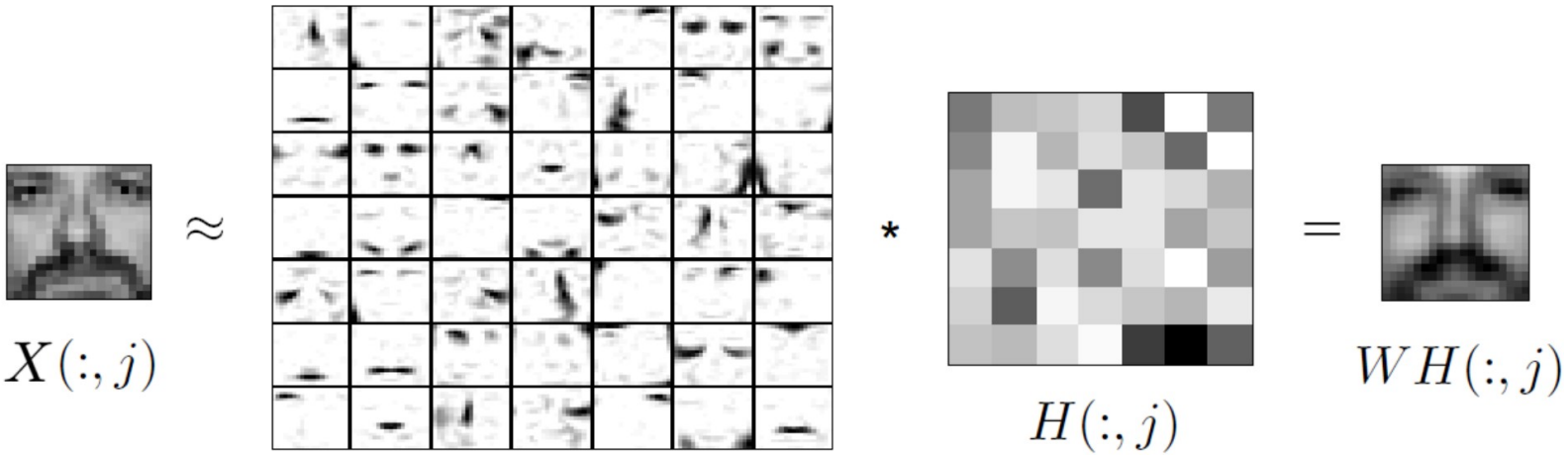
where $R$ is the decomposition rank

- Parts-based representation (Lee, et al., Nature 1999)
- Interpretable linear model
- Works for limited data
- Worse reconstruction than SVD, but different application
- Useful for source separation, clustering, blind hyperspectral unmixing, text mining, etc.
- Very limited research of nonlinear extensions

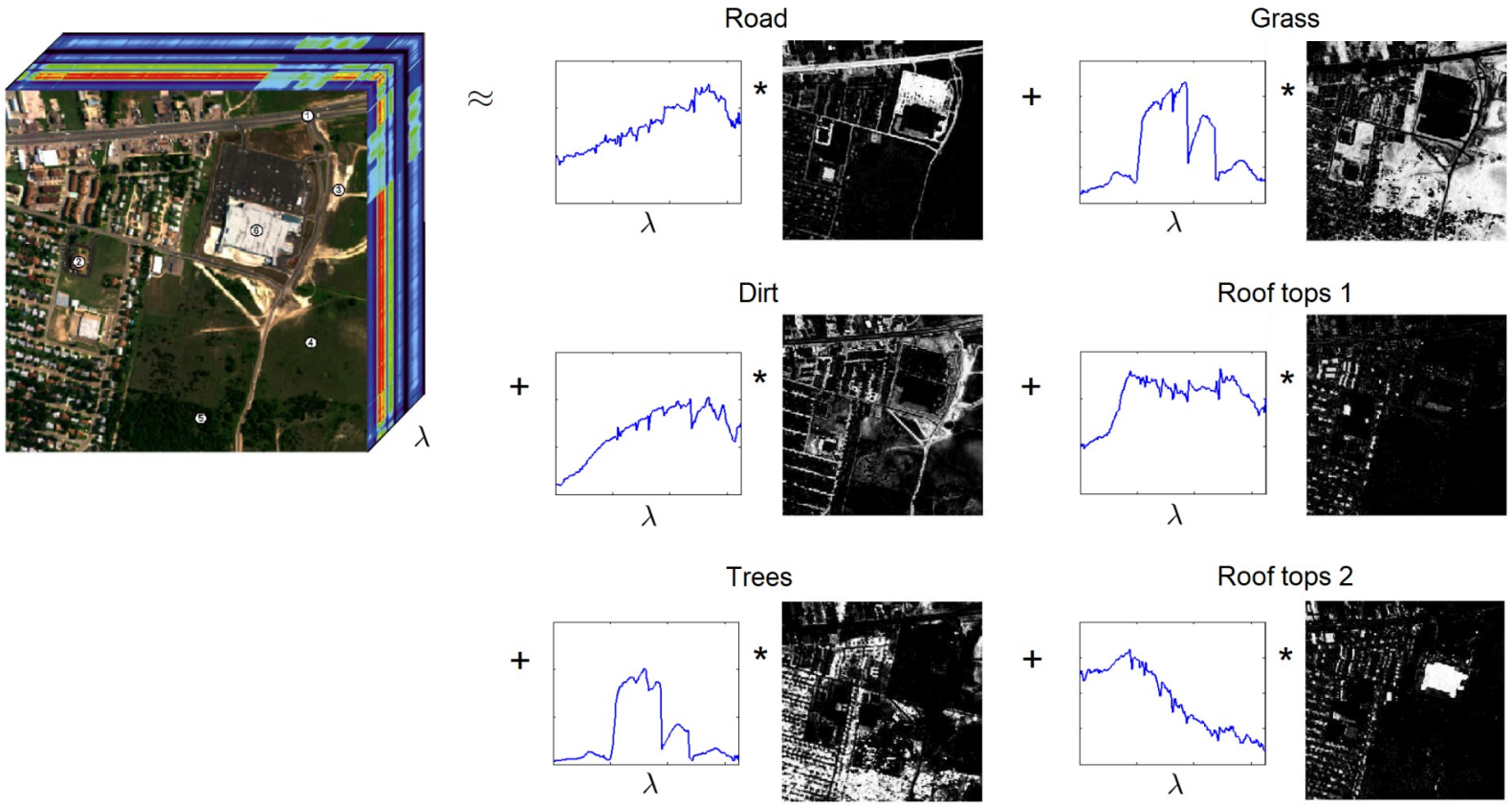
# NMF variants

- NMF, $\boldsymbol{W} \geqslant 0, \boldsymbol{H} \geqslant 0$
- ONMF, $\boldsymbol{W} \geqslant 0, \boldsymbol{H} \geqslant 0, \boldsymbol{W}\boldsymbol{W}^T = I$ and/or $\boldsymbol{H}\boldsymbol{H}^T = I$
- semi-NMF, $\boldsymbol{H} \geqslant 0$
- sparse NMF, $\boldsymbol{W} \geqslant 0, \boldsymbol{H} \geqslant 0, \boldsymbol{W}$ and/or $\boldsymbol{H}$ are sparse
- NMU, $\boldsymbol{W}\boldsymbol{H} \leqslant \boldsymbol{X}, \boldsymbol{W} \geqslant 0, \boldsymbol{H} \geqslant 0$
- deep NMF, $\boldsymbol{X} \approx \boldsymbol{W}\boldsymbol{H}_1\boldsymbol{H}_2 \ldots \boldsymbol{H}_t, \boldsymbol{W} \geqslant 0, \boldsymbol{H}_i \geqslant 0$ for all $i$
- convolutive NMF
- kernel NMF
- complex NMF
- ...

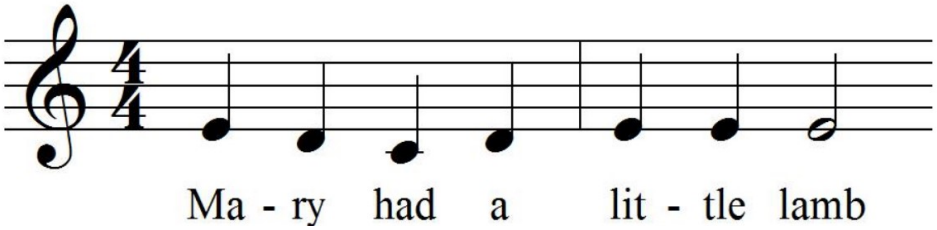
# Features extraction from images

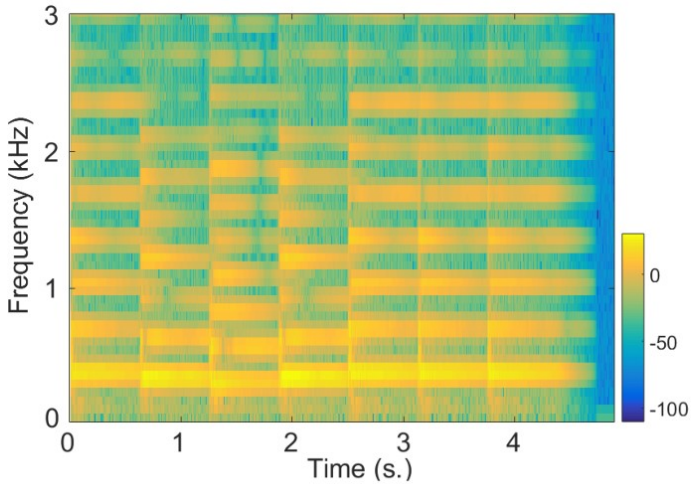


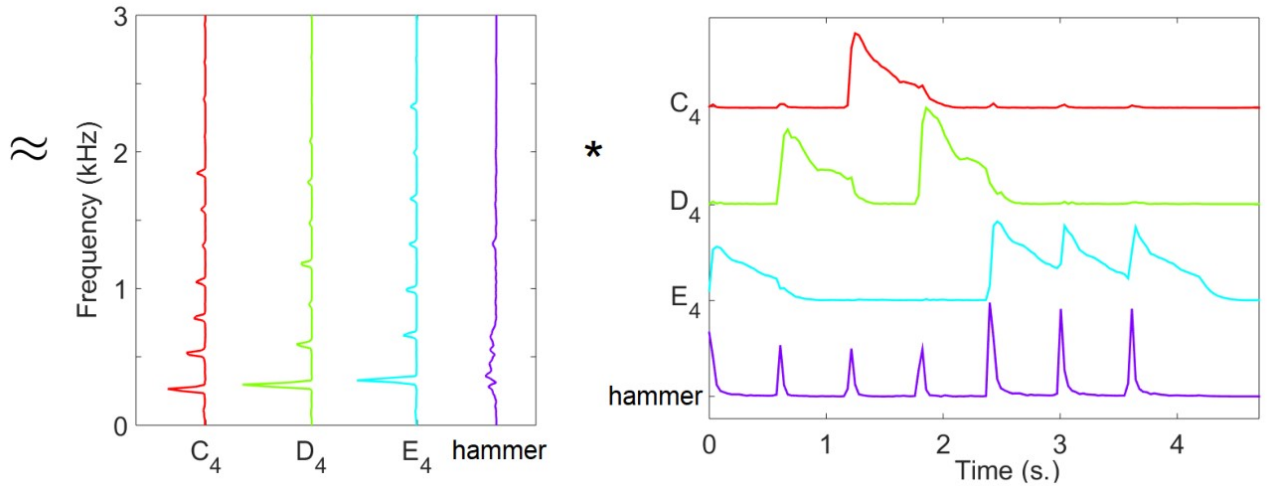
# Blind hyperspectral unmixing



# Source separation

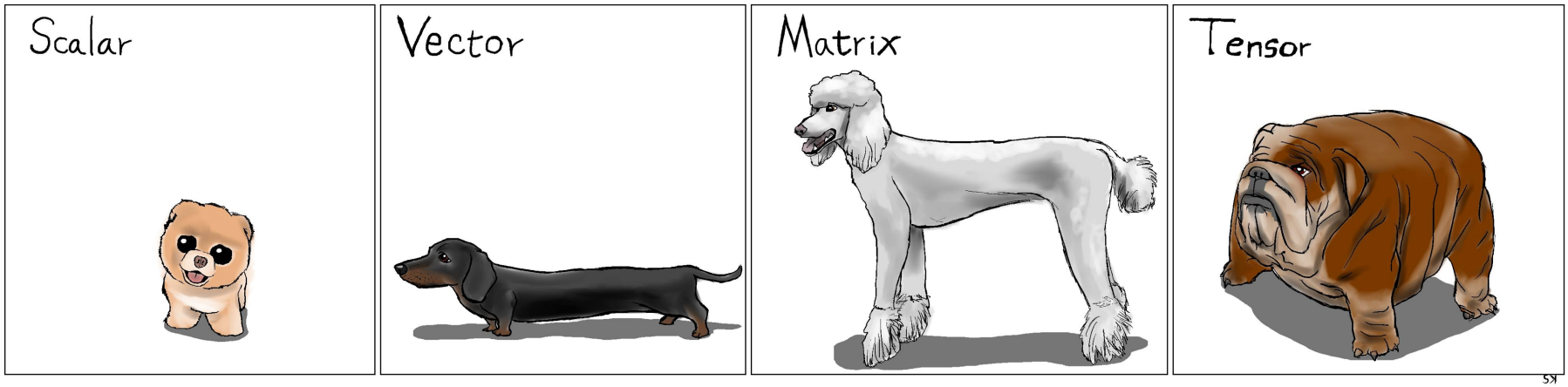


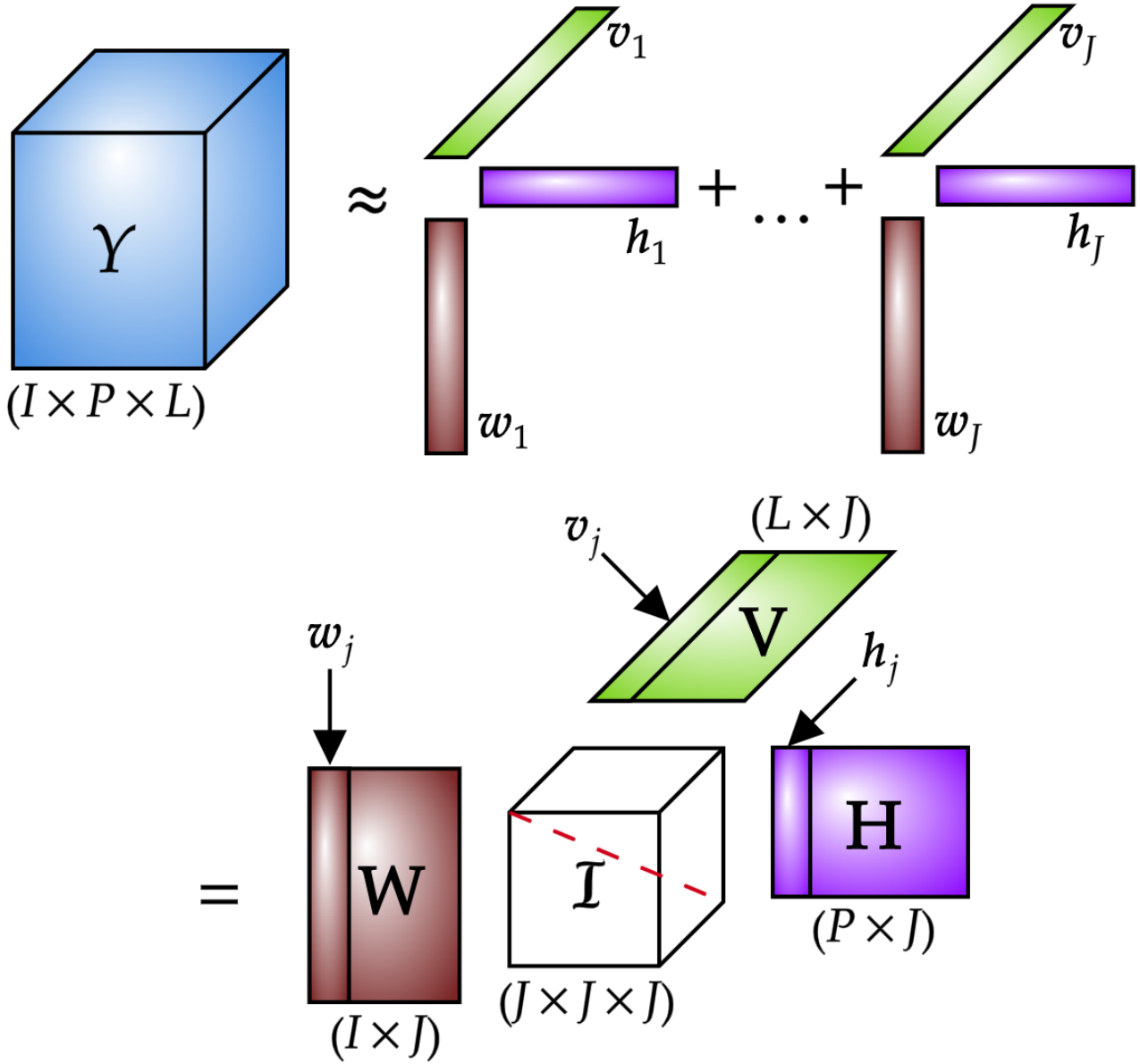



# Tensors

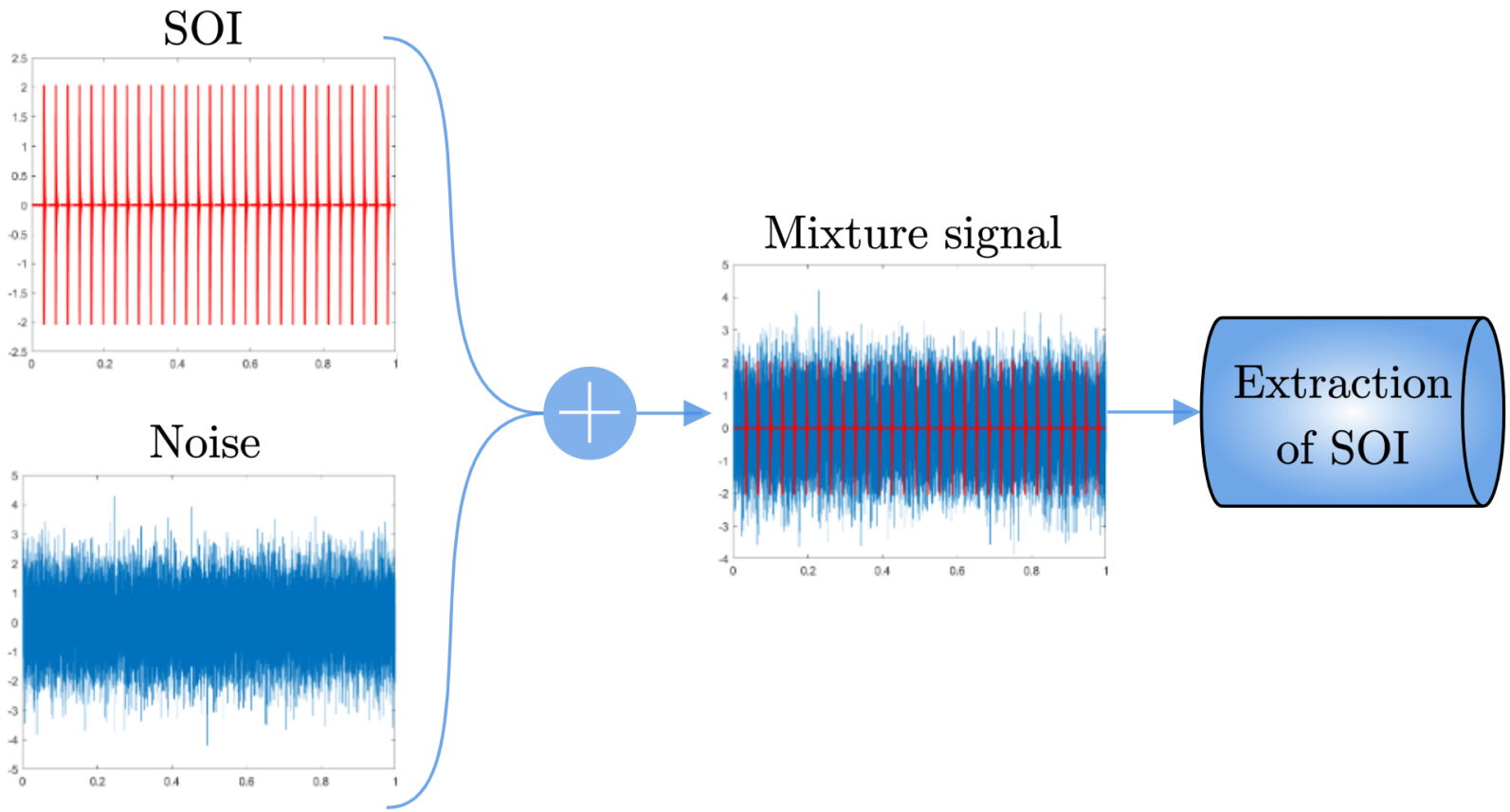
- Generalization of vector, matrix
- Muli-linear algebra
- The dimension is mostly called "order/mode"
- RGB images are 3-mode tensors, videos are 4-mode tensors, weights of convolutional layers are 4-th mode tensors
- Many decomposition techniques



# Nonnegative tensor factorization



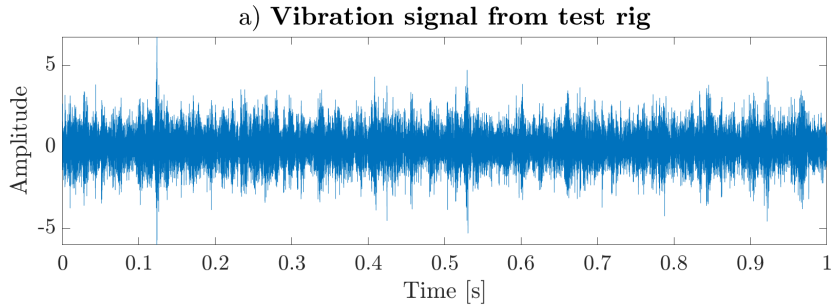
# Fault detection in bearings using NMF and NTF - problem formulation

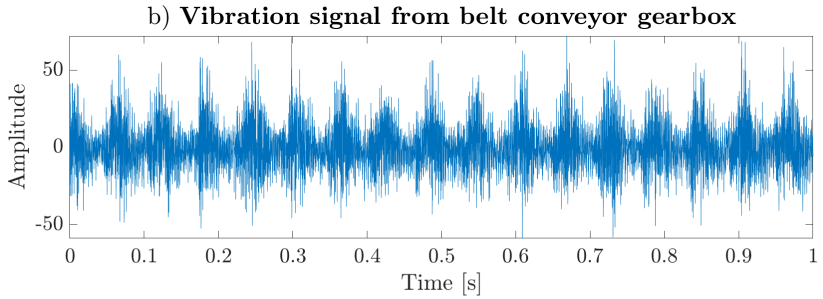


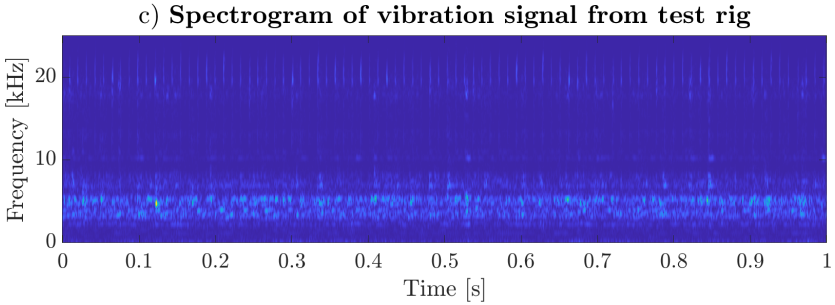
# Fault detection in bearings using NMF and NTF - experimental setup

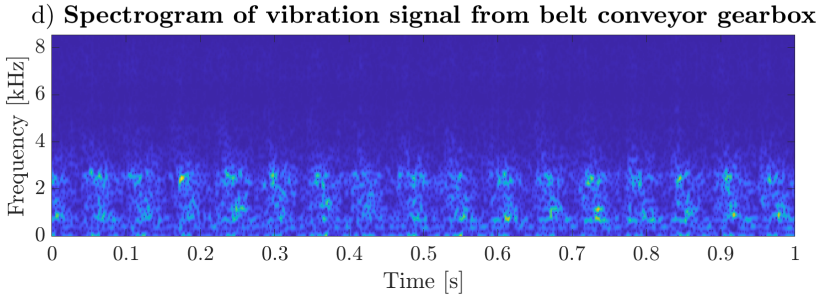
- 3 cost functions for both NMF/NTF: Euclidean, Kullback-Leibler and Itakura-Saito
- 25 simulated signals with different SNRs
- 2 real vibration signals: signal from test rig and signal from belt conveyor gearbox
- Evaluation is done visually and via spectral based indicator (SBI)
- Gabor, M., Zdunek, R., Zimroz, R., Wodecki, J., Wylomanska, A.: "Non-negative tensor factorization for vibration-based local damage detection". *Mechanical Systems and Signal Processing* 2023

# Fault detection in bearings using NMF and NTF - real signals

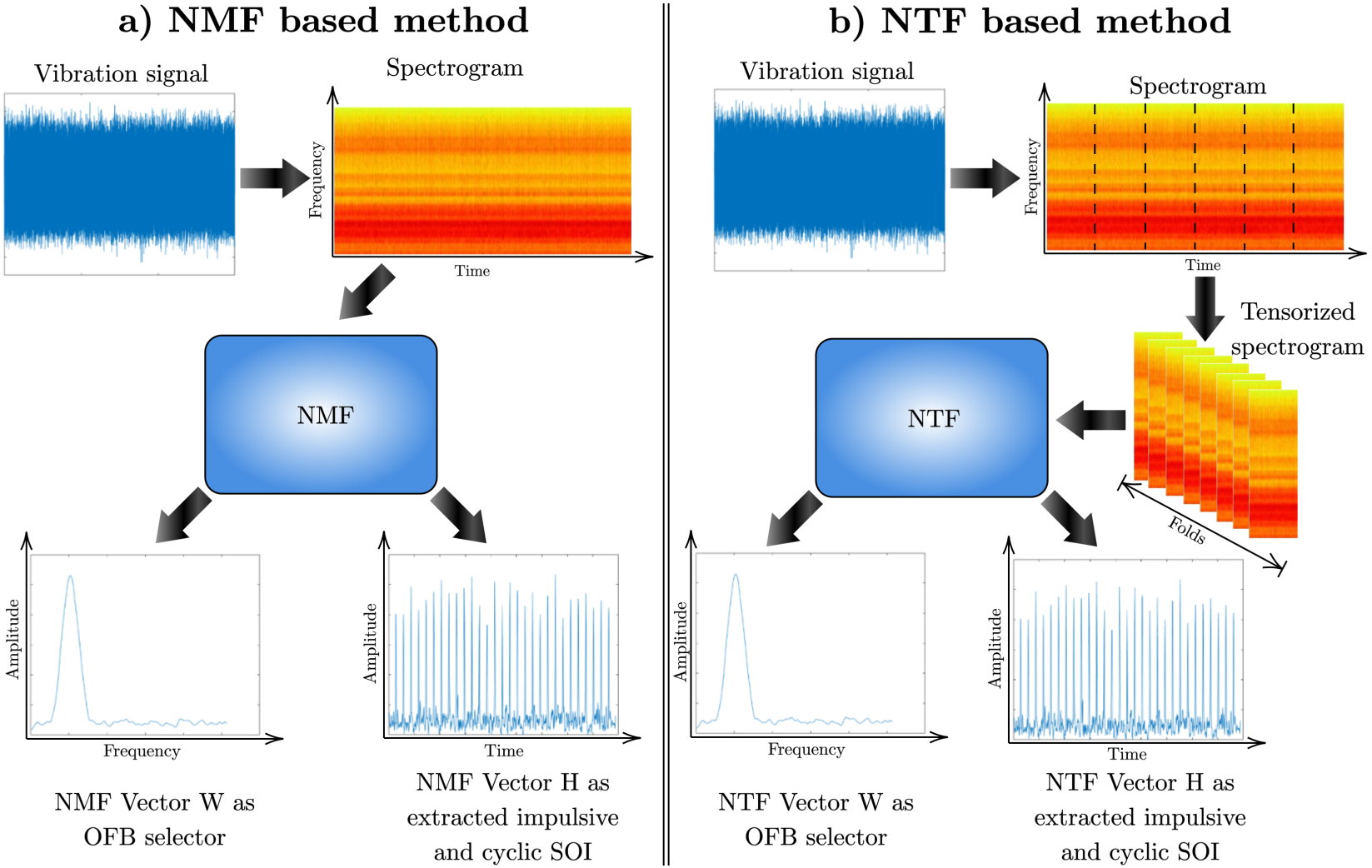


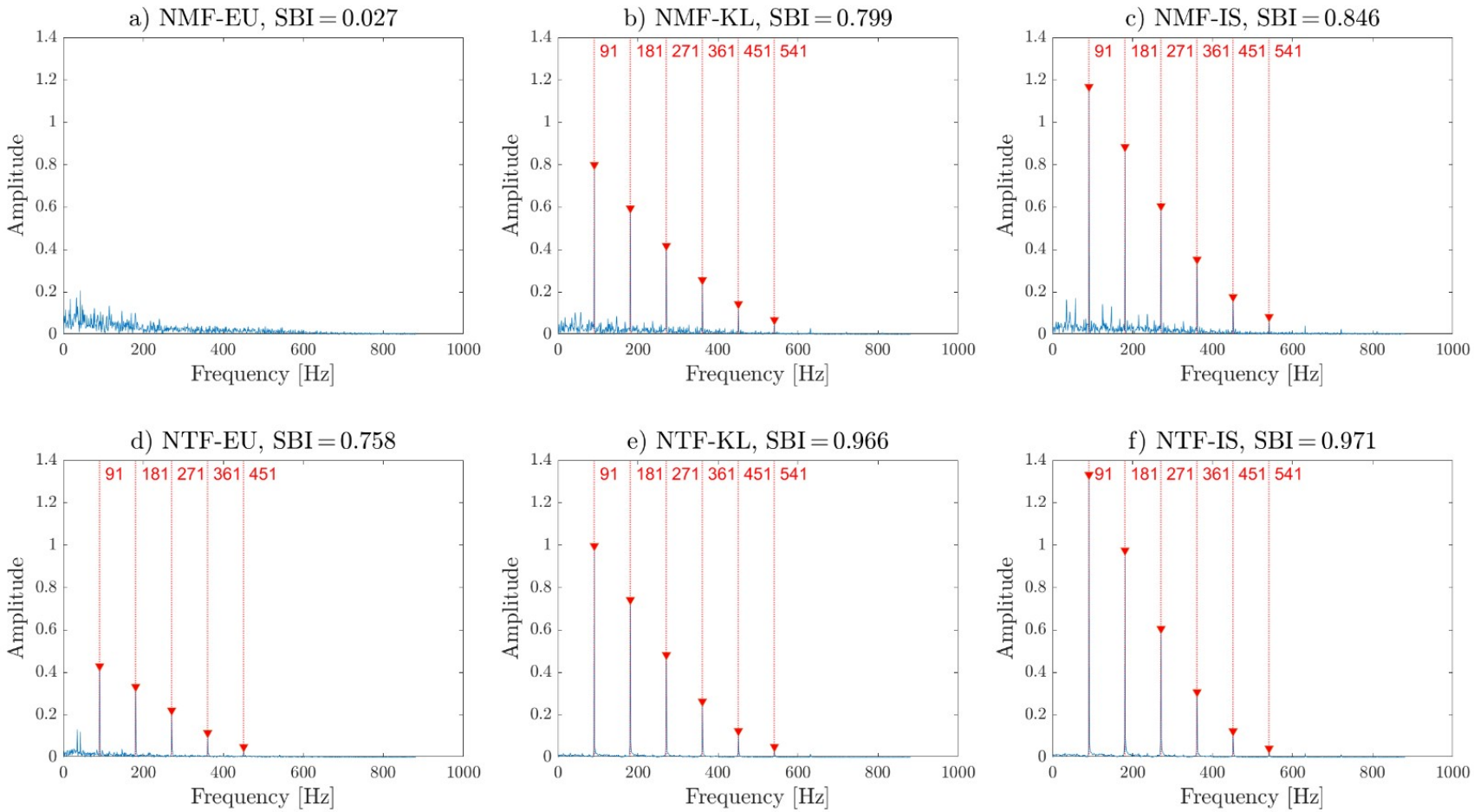



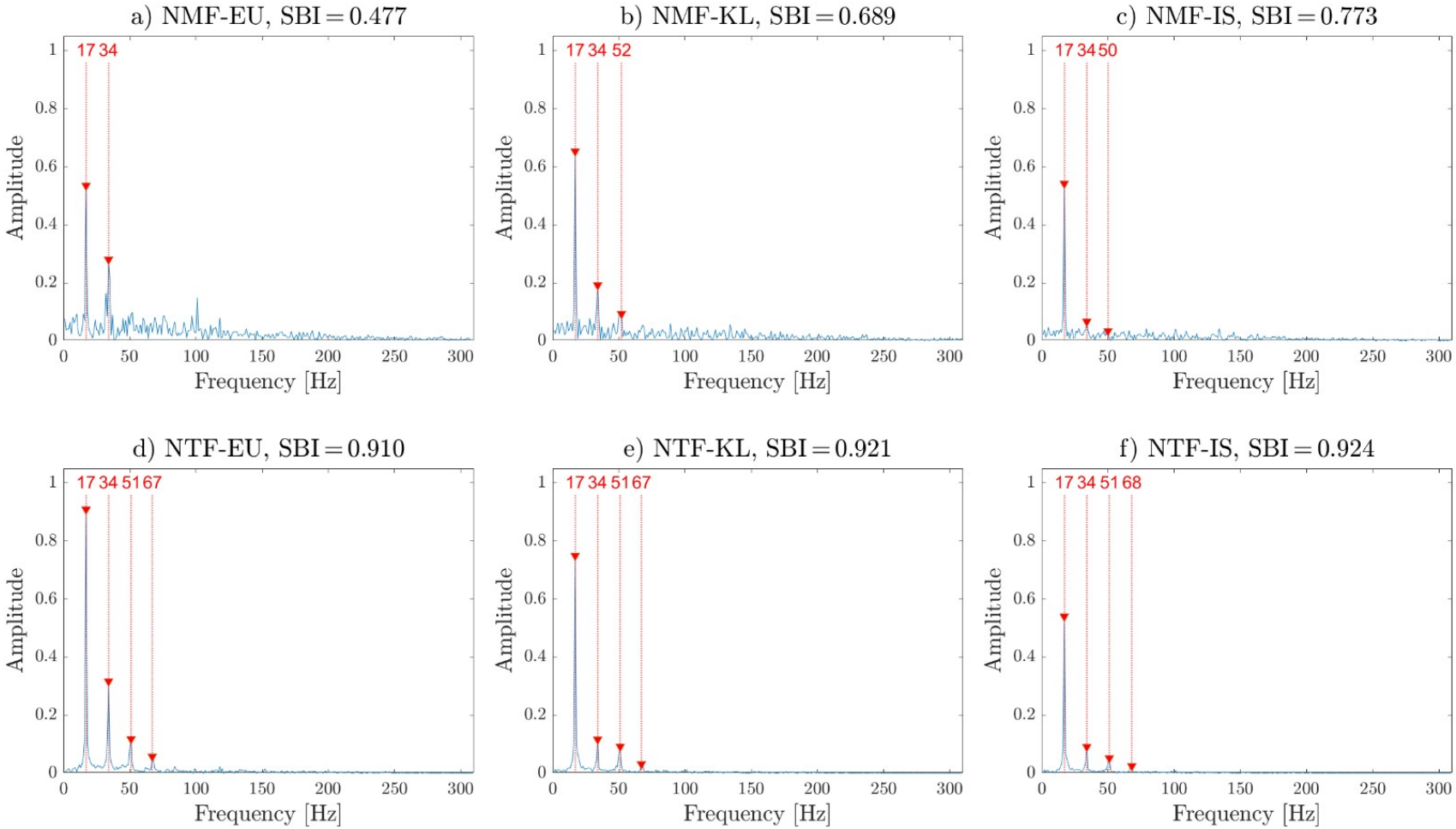
# Fault detection in bearings using NMF and NTF - method



# Fault detection in bearings using NMF and NTF - test rig results



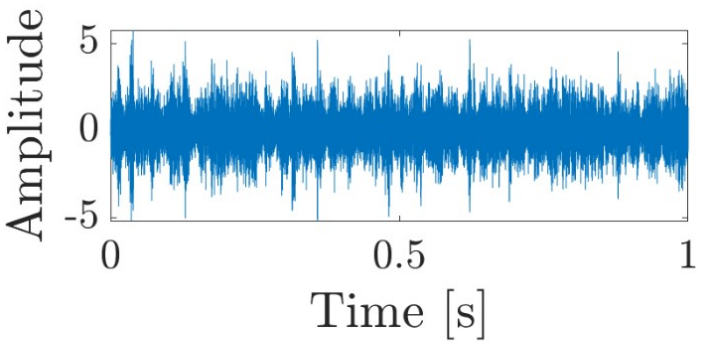
# Fault detection in bearings using NMF and NTF - belt conveyor results



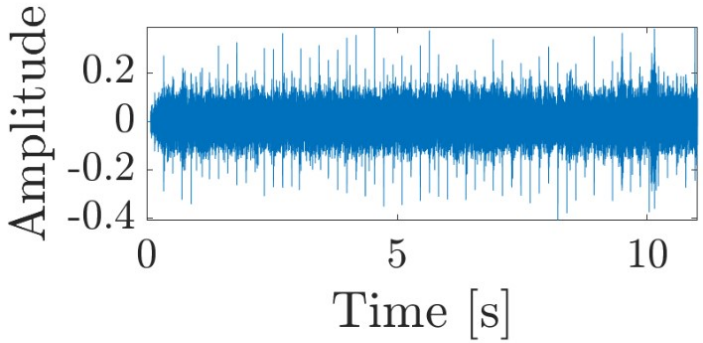
# Orthogonal NMF as optimal filter selector - experimental setup

- 3 simulated signals for Gaussian noise case and six simulated signals with non-Gaussian noise
- 2 real signals: vibration signal from test rig and acoustic signal from belt convey or idler
- Evaluation is done by kurtosis (Gaussian noise case) and envelope-spectrum based indicator (ENVSI)
- Gabor, M., Zdunek, R., Zimroz, R., Wylomanska, A.: "Bearing damage detection with orthogonal and non-negative low-rank feature extraction". IEEE Transactions on Industrial Informatics - under review

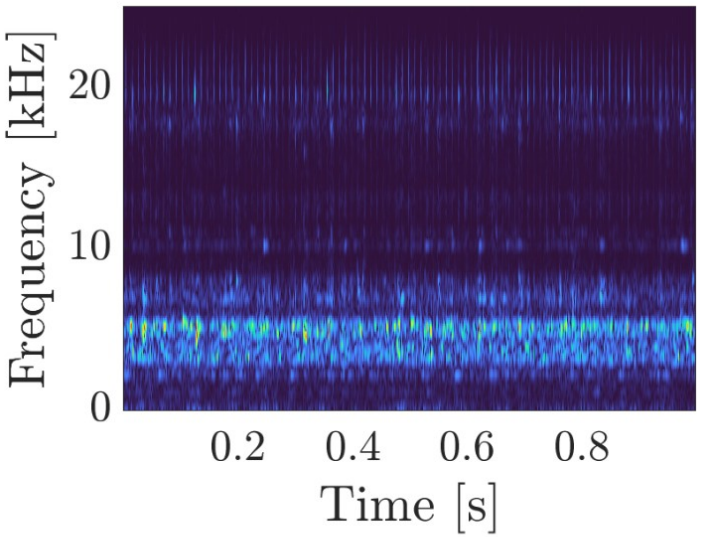
# Orthogonal NMF as optimal filter selector - real signals



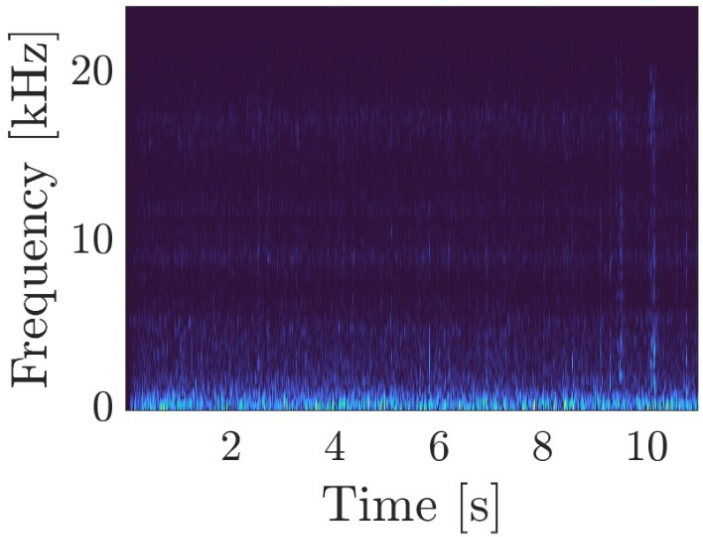

(a) Gaussian noise




(b) non-Gaussian noise

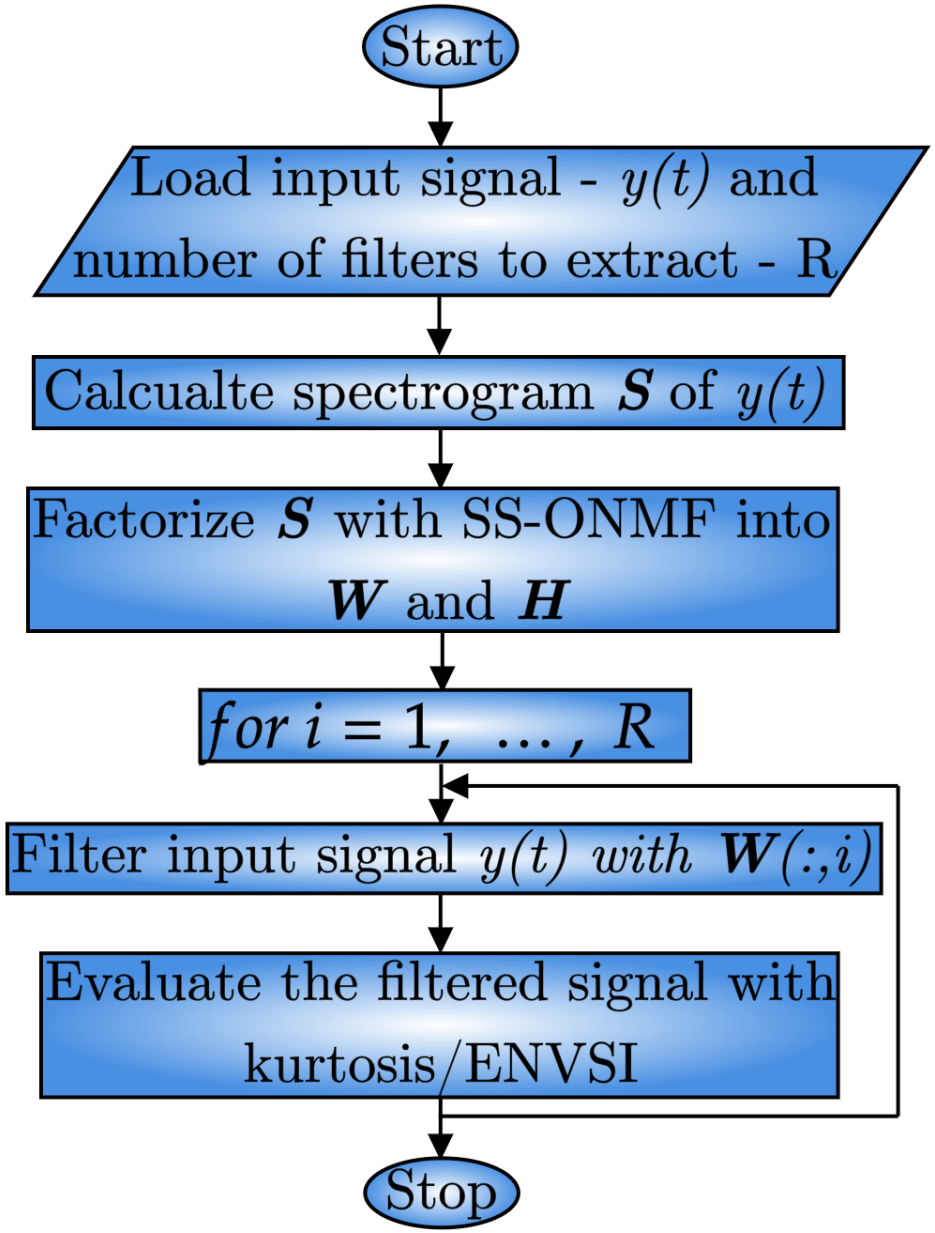



(c) Gaussian noise

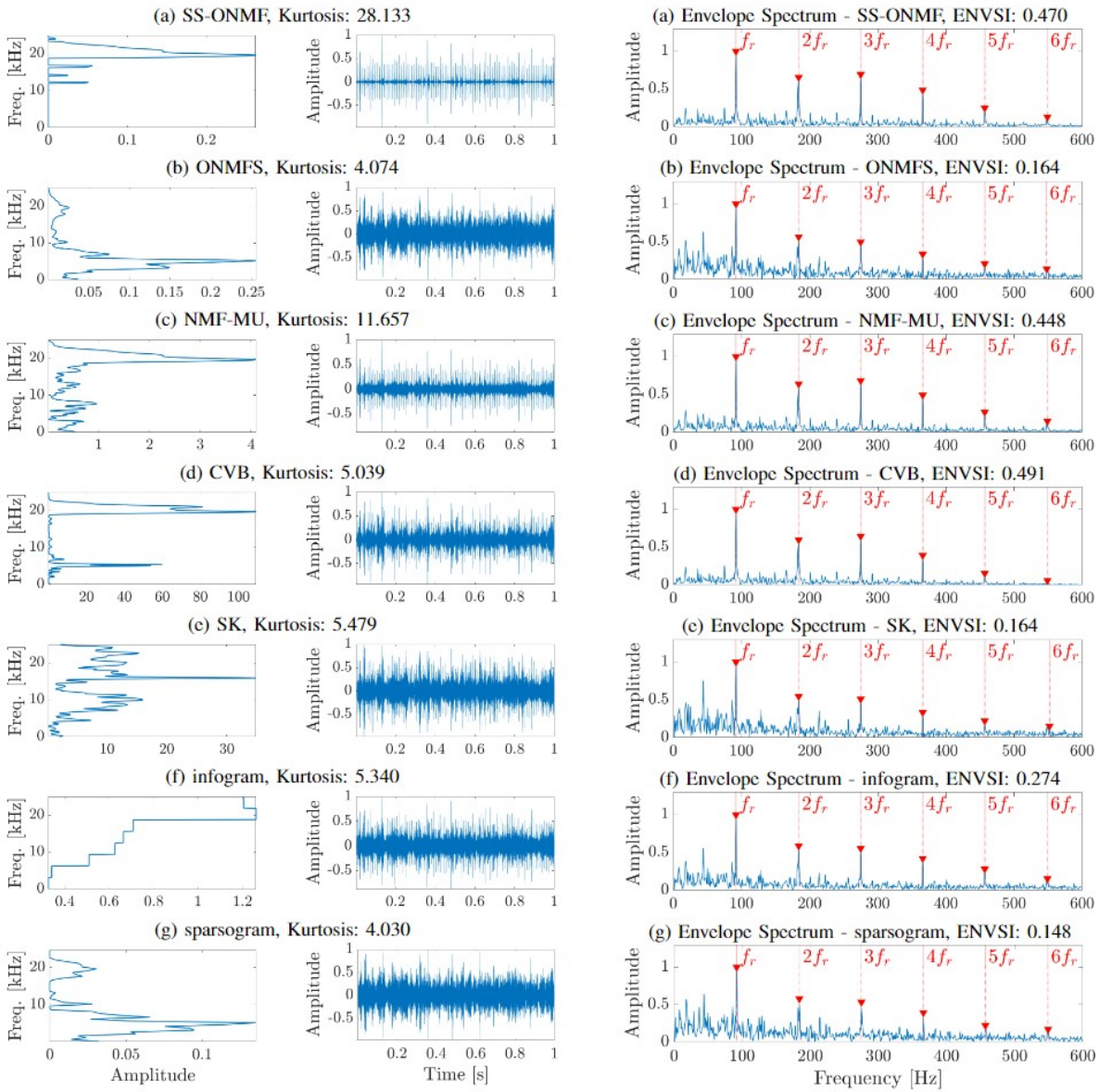



(d) non-Gaussian noise

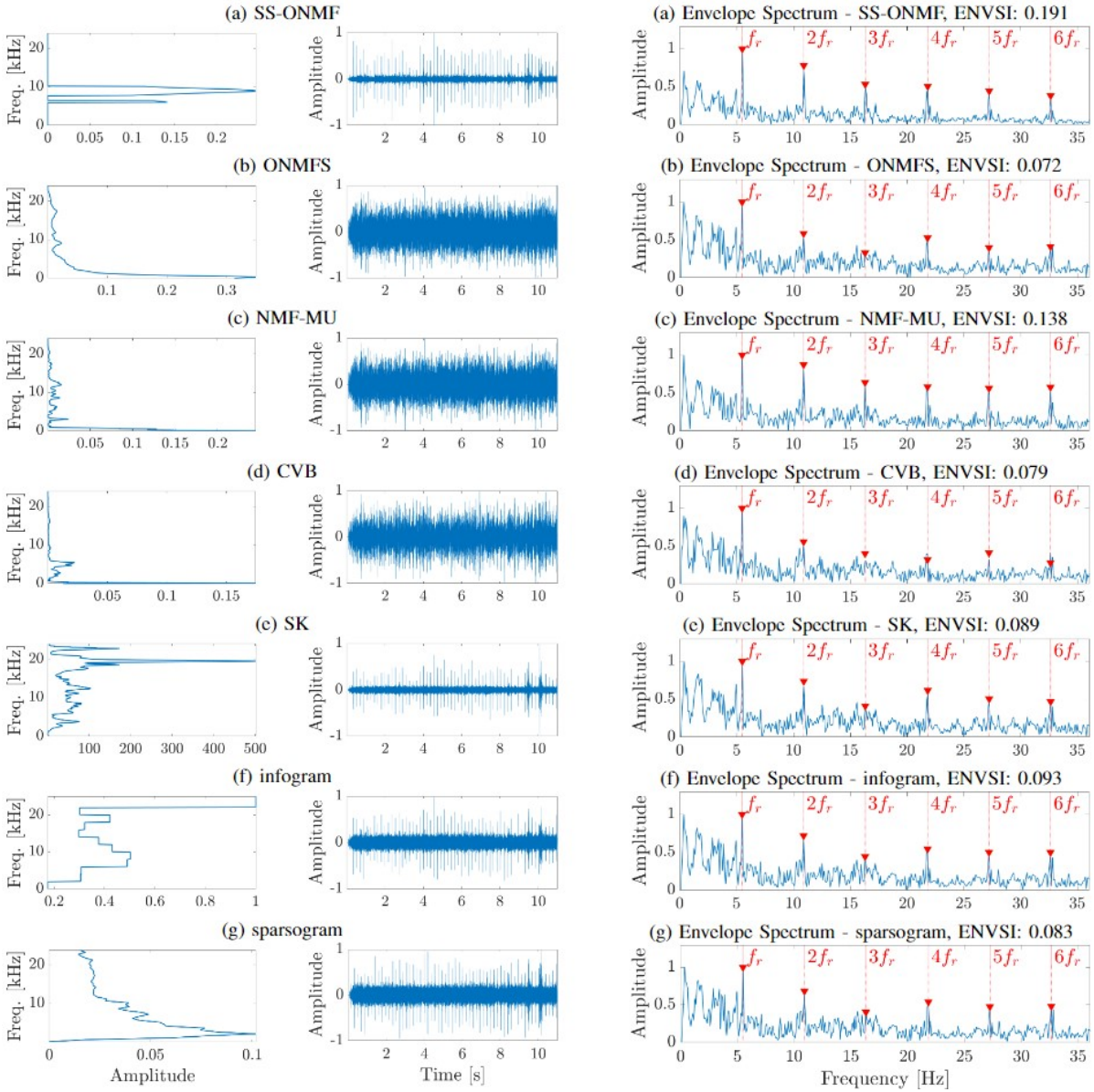
# Orthogonal NMF as optimal filter selector - method



# Orthogonal NMF as optimal filter selector - Gaussian noise results



# Orthogonal NMF as optimal filter selector - non-Gaussian noise results



# Future works

- Multichannel NMF - current work
- Sparse non-negative matrix underapproximation as informative frequency band selector
- Projective NMF with Itakura-Saito
- Nonlinear extensions fo NMF/NTF

# The end

Thank you for your attention!