



MONITORING of large scale complex technological systems (European MOIRA project)

Effective Identification of Cyclic Excitation and Resonance in Non-stationary Gearbox Vibration Monitoring



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Subject of the thesis:

Improving virtual sensing by multicomplexity models

Context:

Monitoring of heterogeneous fleet of machines, using a generic model, and solving an inverse identification problem

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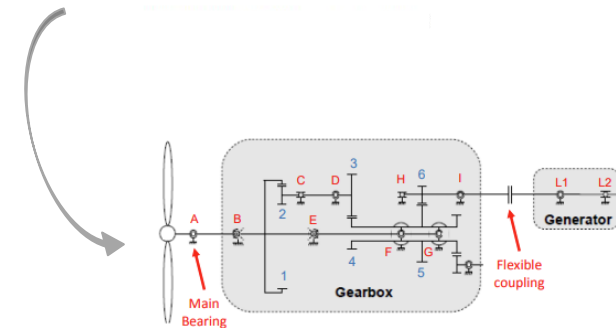
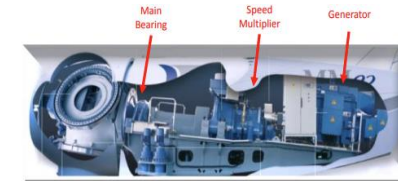
Introduction:

- Physical Versus Virtual Sensors
- Heterogeneous Fleet of Machines



Effective Identification of Cyclic and Resonance Excitations:

- Introduction to the identification approaches
- Description of the Problem and the system modeling
- Identification results





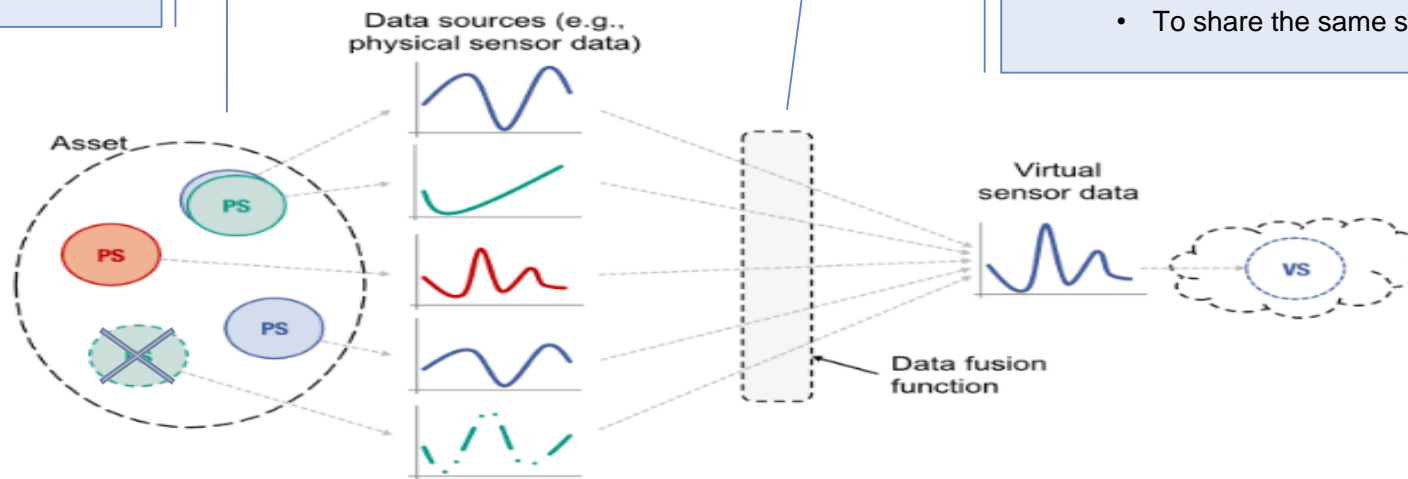
Physical Versus Virtual Sensors:

Drawbacks:

- Limited spatial and temporal coverage
- Uncertainty
- Limited robustness
- Accuracy lost over time

Advantages:

- Significantly lower costs
- Where physical sensors can not be deployed
- Reducing signal noise
- Can recognize and compensate drift phenomenon in physical sensors
- Flexible and can be redesigned as required
- To share the same signal along the fleet of machines



Virtual Sensor Concept,

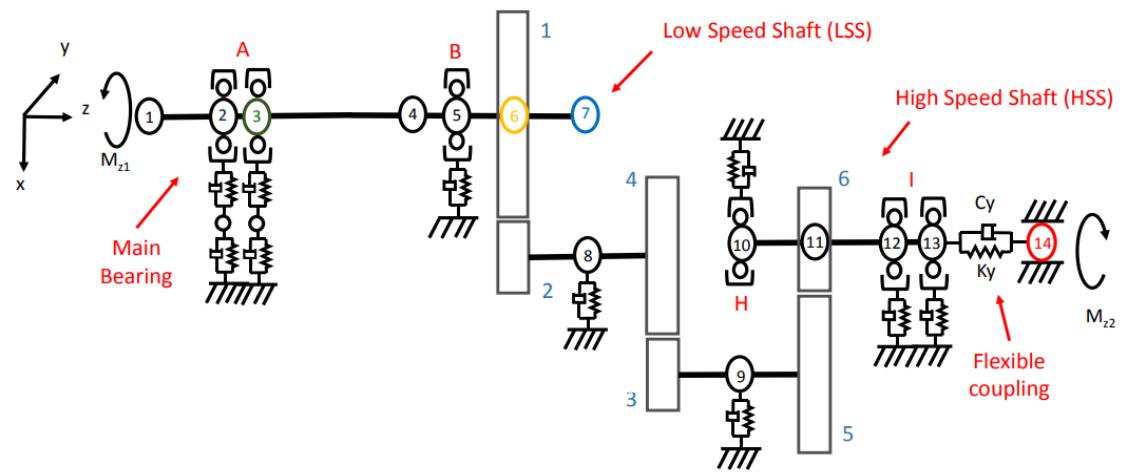
Martin D, Kühl N, Satzger G. *Virtual sensors*. Business & Information Systems Engineering. 2021 Jun;63:315-23.



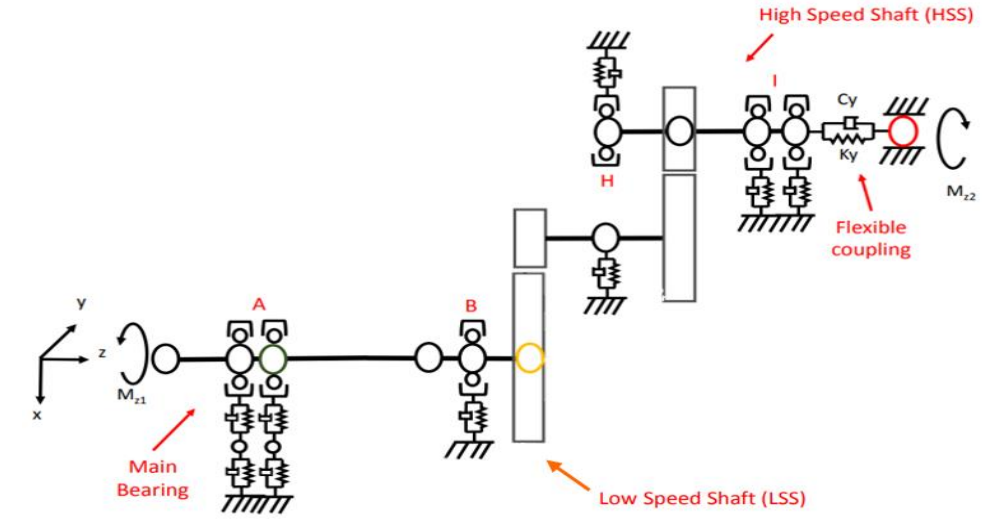
Heterogeneous Fleet of Machines

Different Architecture Types

3-Stage Gear Box



2-Stage Gear Box



Bearings	Characteristics
A	Double row spherical. Self-aligning
B	Single row tapered
C	Double row cylindrical

Bearings	Characteristics
D	Double row tapered. Self-aligning
E	Single row tapered
F/G	Single row cylindrical

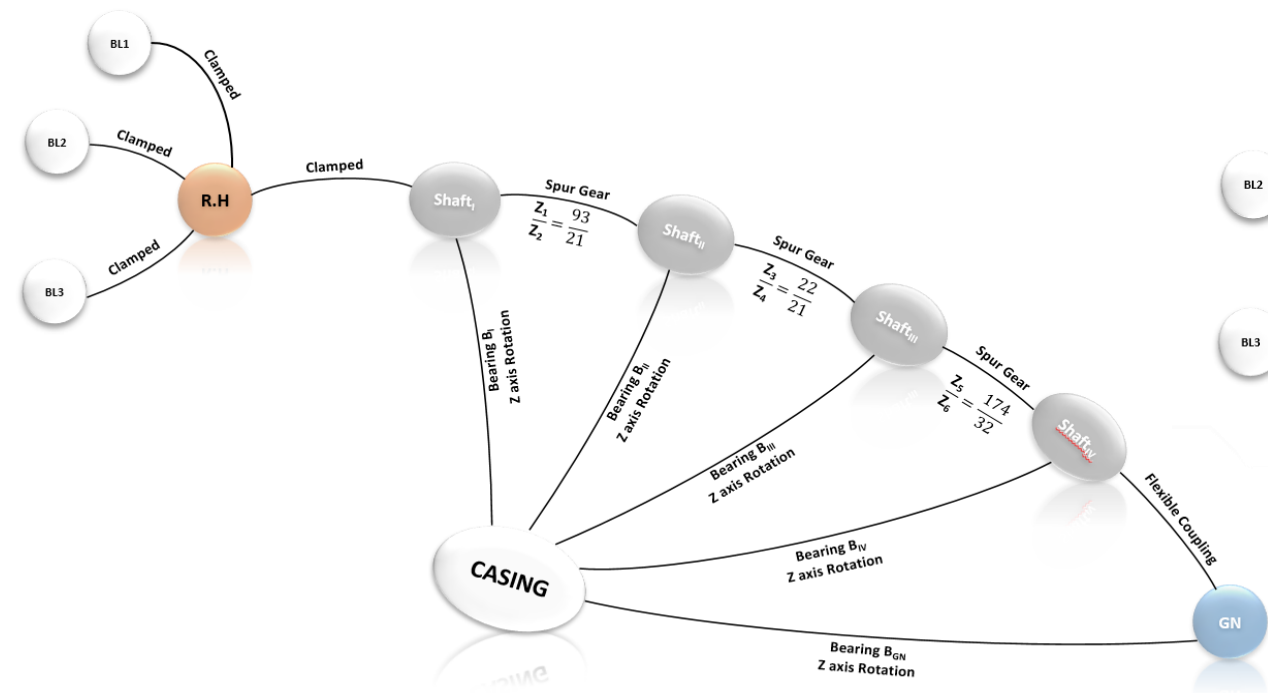
Bearings	Characteristics
H	Single row tapered
I	Double row tapered
L1 /L2	Deep groove ball bearings



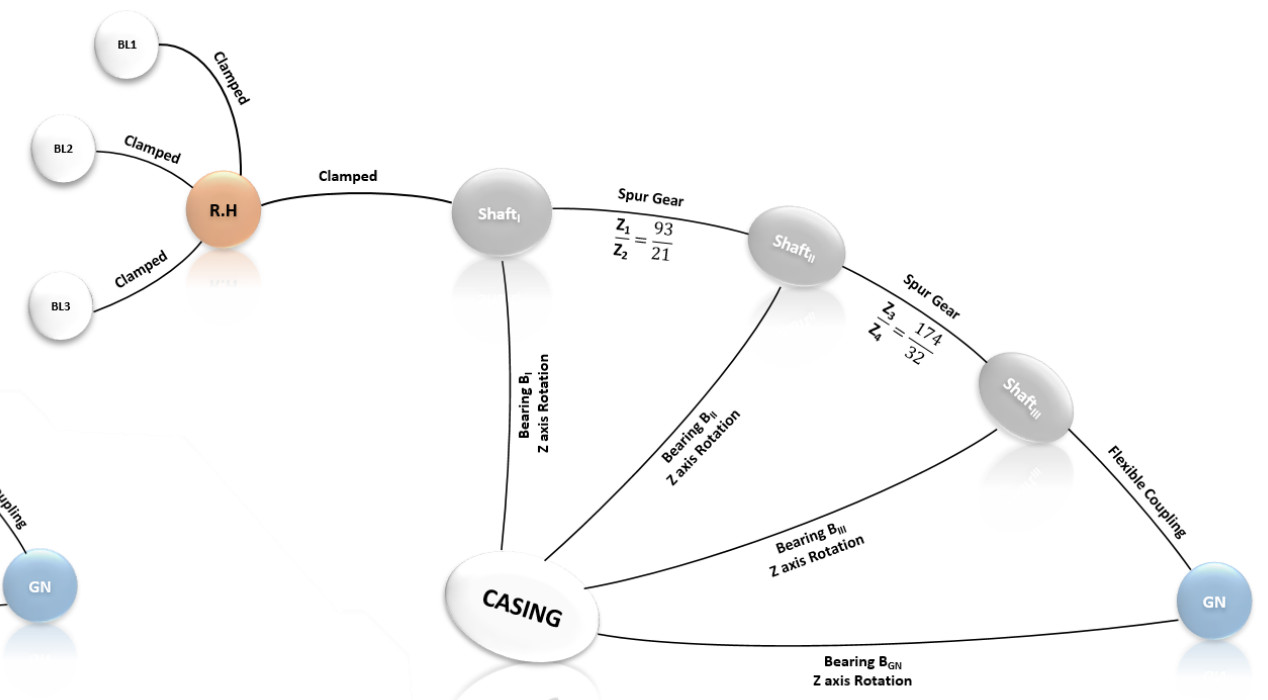
Graph Theory:

Layer 1 - Geometry

3-Stage Gear Box

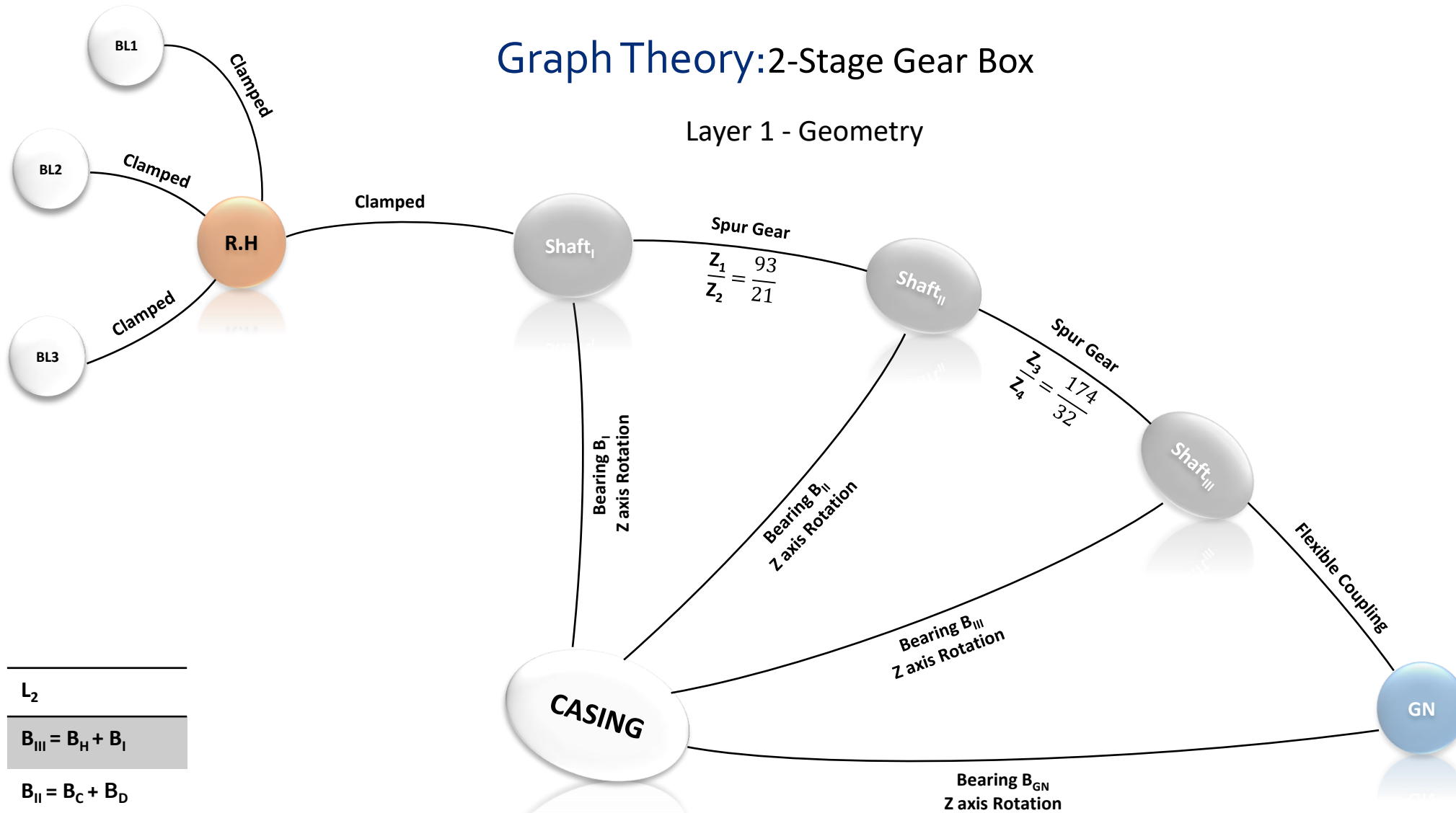


2-Stage Gear Box



Graph Theory: 2-Stage Gear Box

Layer 1 - Geometry



R.H: Rotating Hub
 Bl: Blade
 Sh: Shaft
 GN: Generator

Gear	Z: Teeth Number
1	93
2	21
3	174
4	32

L_2

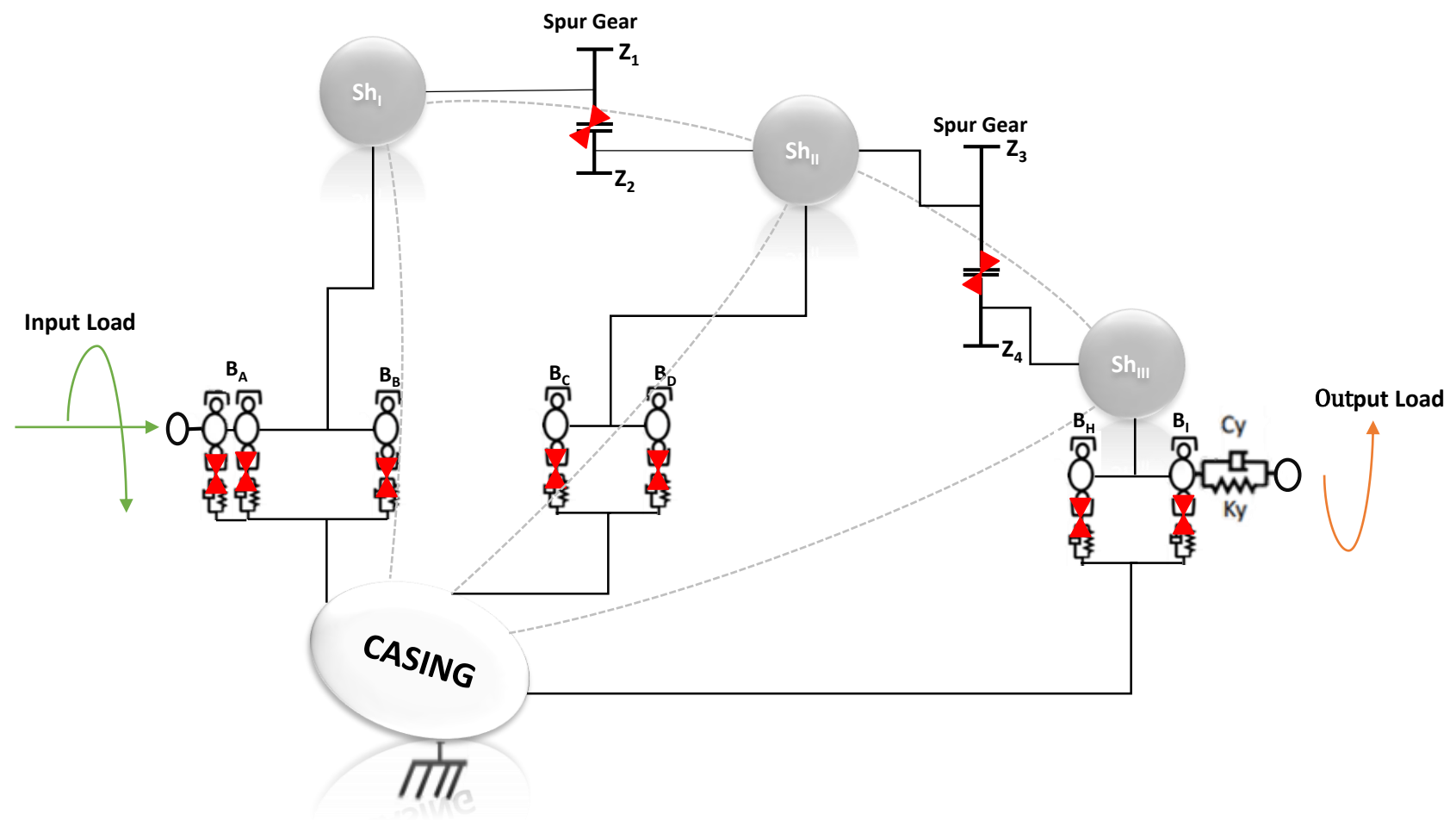
$B_{III} = B_H + B_I$

$B_{II} = B_C + B_D$

$B_I = B_A + B_B + B_E$

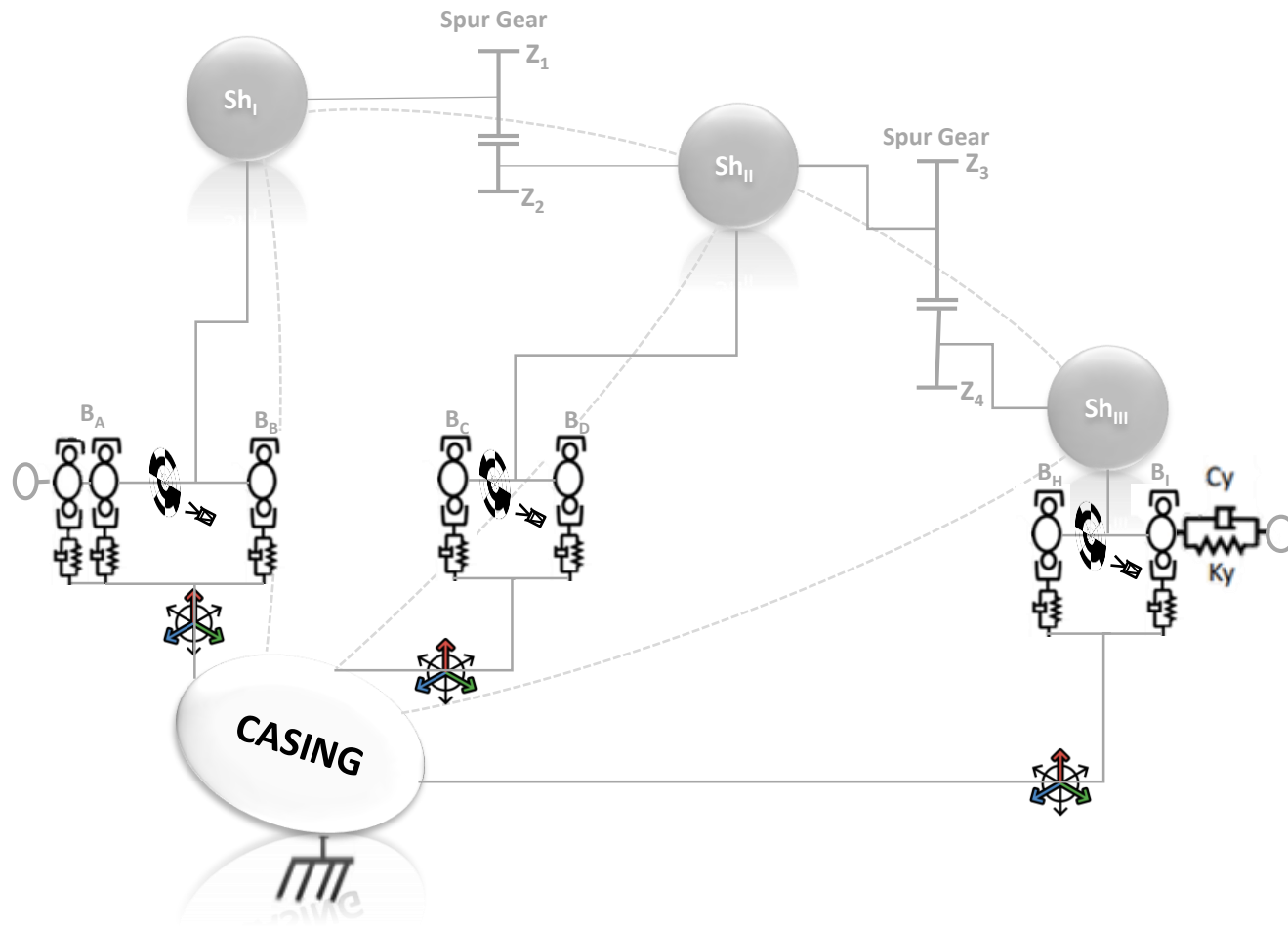
Graph Theory: 2-Stage Gear Box

Layer 2 – Interaction and Boundary conditions



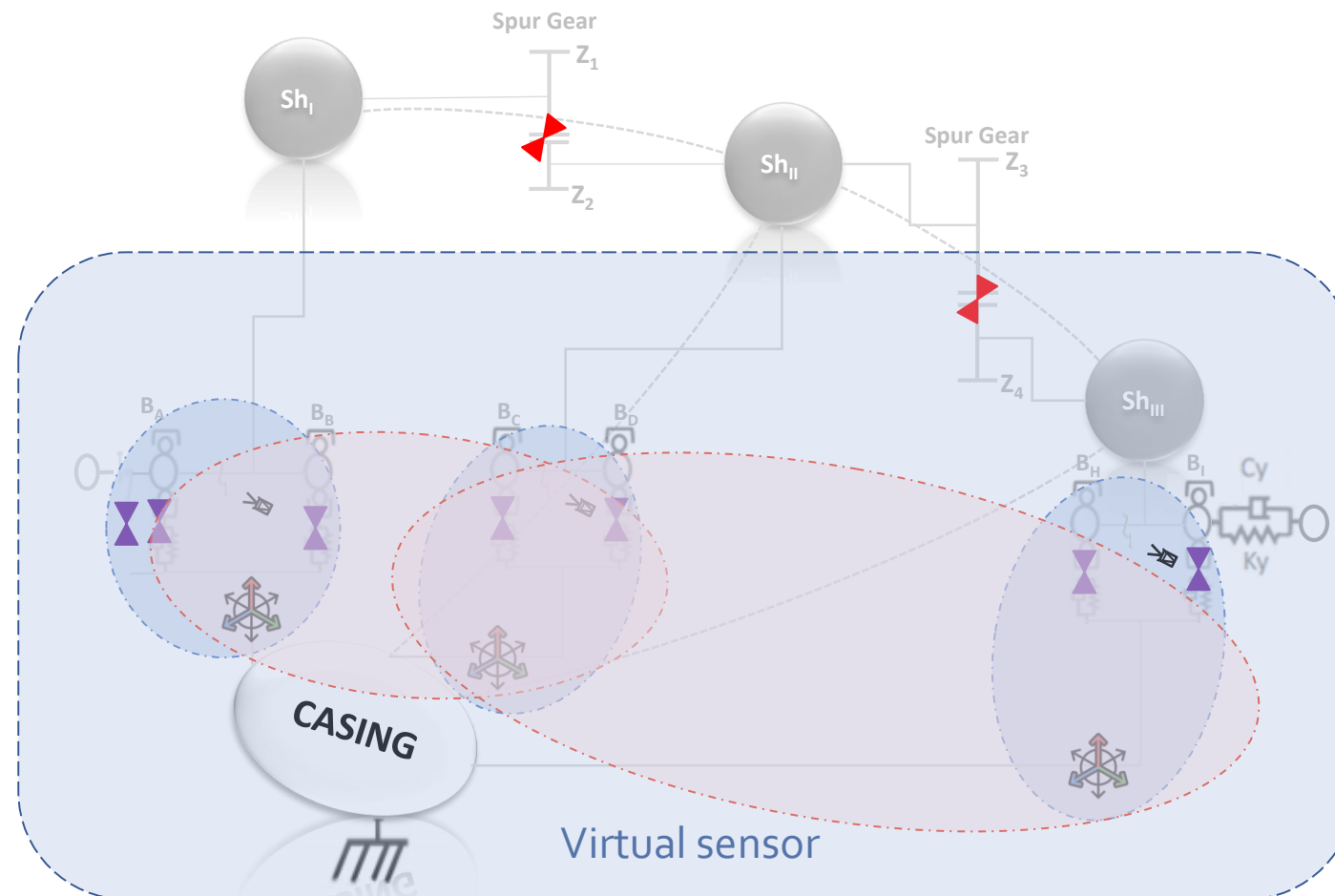
Graph Theory: 2-Stage Gear Box

Layer 3 – Physical Sensors



Graph Theory: 2-Stage Gear Box

Layer 4 – Virtual Sensor



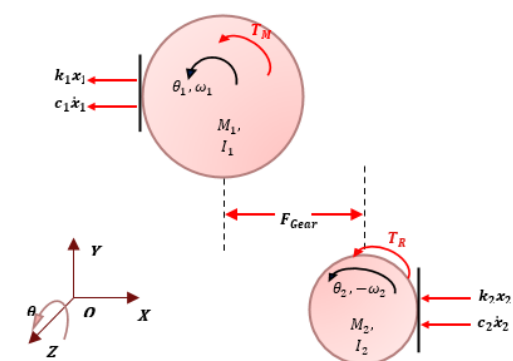
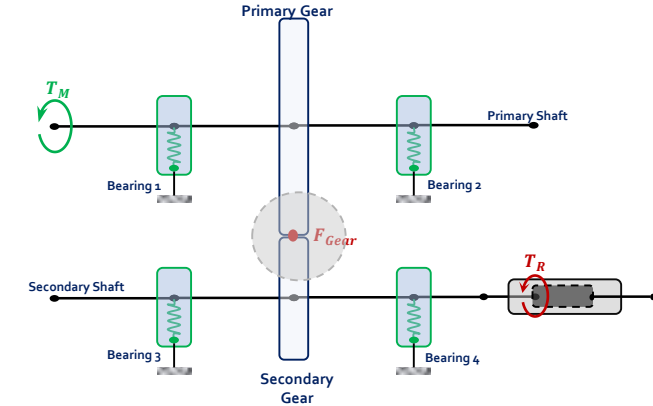
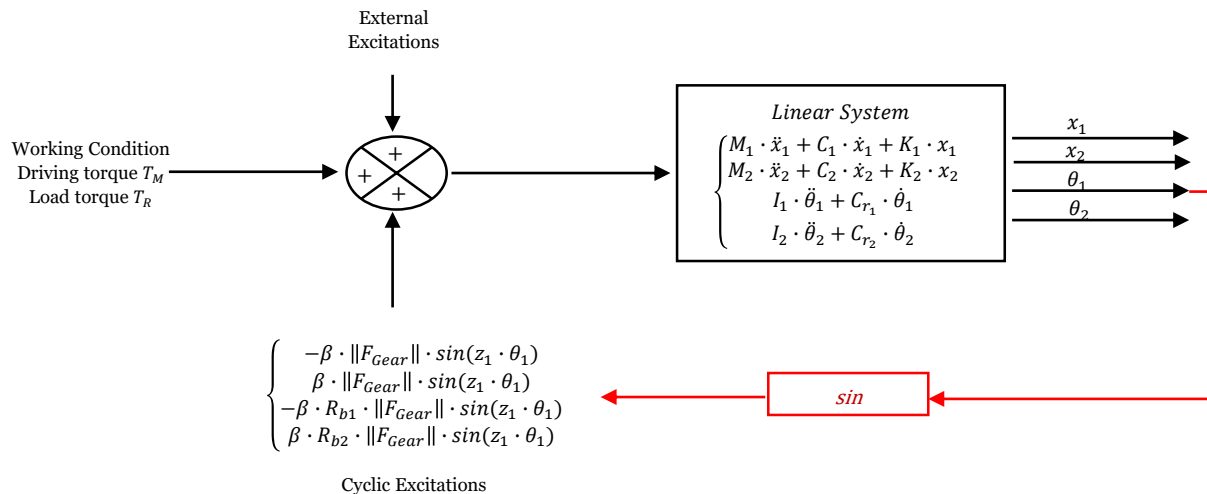
Effective Identification of Cyclic and Resonance Excitations:

$$\begin{cases} M_1 \cdot \ddot{x}_1 + C_1 \cdot \dot{x}_1 + K_1 \cdot x_1 = -F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \\ M_2 \cdot \ddot{x}_2 + C_2 \cdot \dot{x}_2 + K_2 \cdot x_2 = F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \\ I_1 \cdot \ddot{\theta}_1 + C_{r1} \cdot \dot{\theta}_1 = \|T_M\| - R_{b1} \cdot F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) + T_{ex1} \\ I_2 \cdot \ddot{\theta}_2 + C_{r2} \cdot \dot{\theta}_2 = \|T_R\| - R_{b2} \cdot F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \end{cases}$$

$$F_{Gear} = F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \quad \& \quad T_{ex1} = C_R \cdot t + C_S \cdot \sin(\omega(t) \cdot t)$$

Fluctuation

$$F_g = K_g \cdot (R_{b1} \cdot \theta_1 + R_{b2} \cdot \theta_2) + C_g \cdot (\dot{R}_{b1} \cdot \dot{\theta}_1 + \dot{R}_{b2} \cdot \dot{\theta}_2) + K_g \cdot (x_1 - x_2) + C_g \cdot (\dot{x}_1 - \dot{x}_2)$$



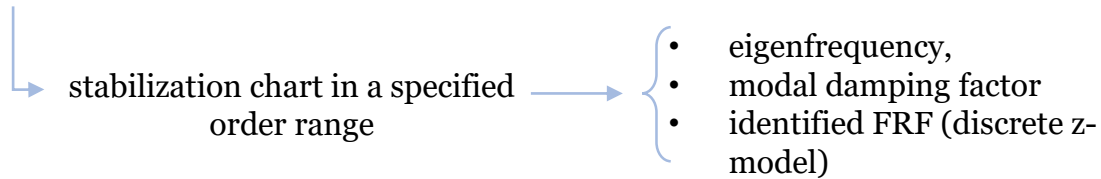


➤ Modal Parameters Identification in Frequency Domain

Algorithm using **complex Frequency Response Function (FRF)** of a **MIMO** system

Based on :

- the Least Square Complex Frequency Estimator (LSCF)



- The Least-Squares Frequency-Domain estimator (LSFD)

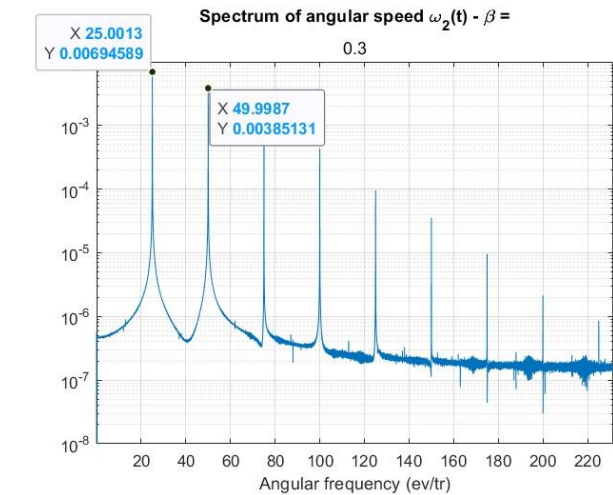
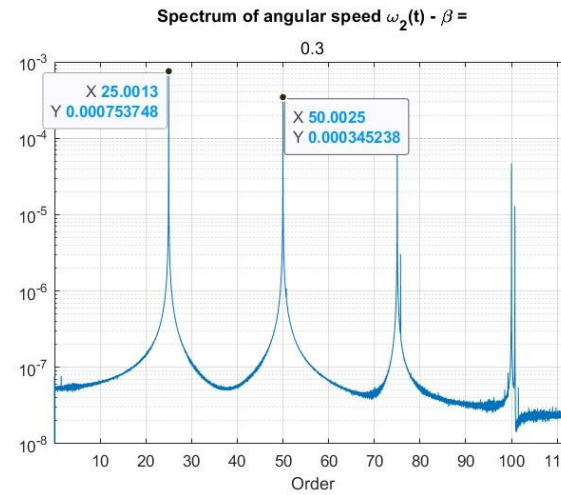
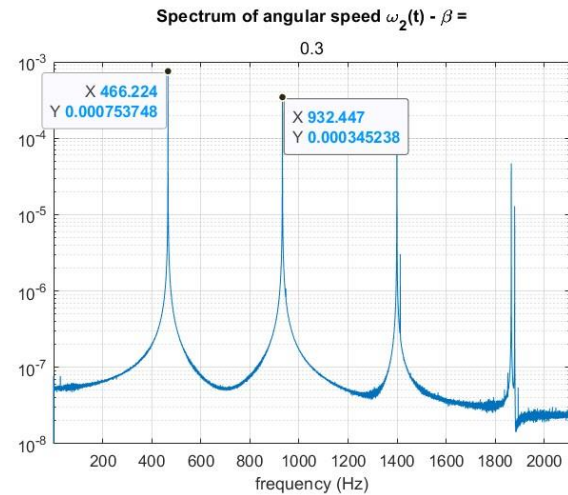
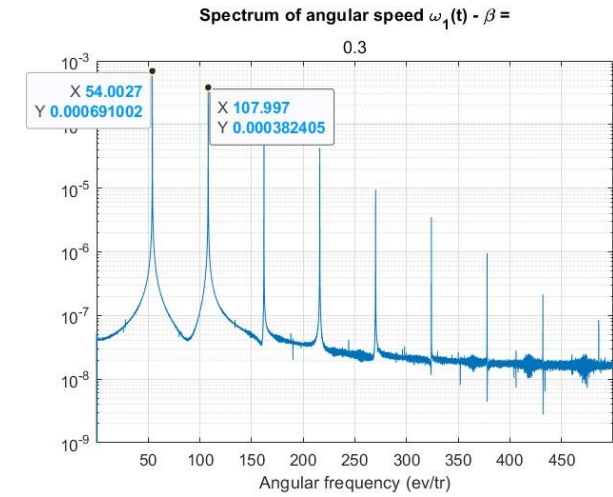
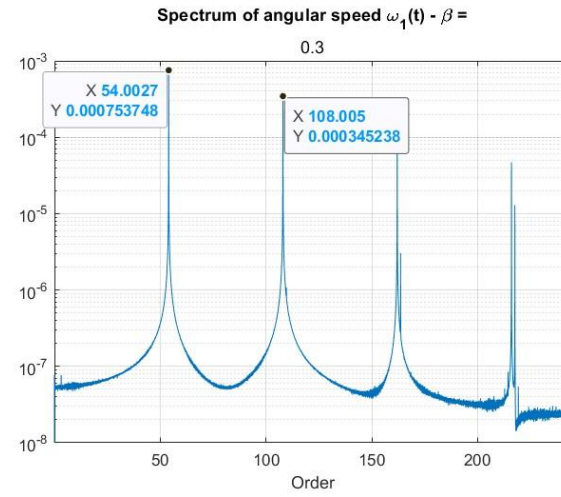
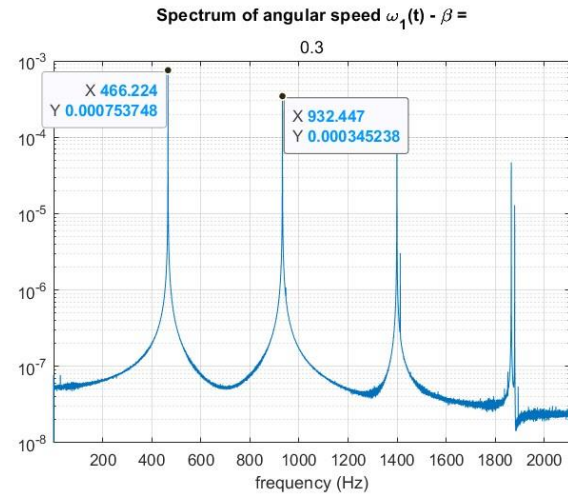


Advantages :

- produces “fast stabilizing” stabilization charts:
- use of **frequency-dependent weighting functions** (the inclusion of weights in the Least Squares cost function allows to improve accuracy of the estimates)
- the LSCF estimator can easily be adapted to more sophisticated solvers such as the Generalized Total Least-Squares implementation



Spectrums in Hz, Order, and Angular (Constant average speed):





Swept frequency Excitation:

Notations:

- $f_i(t)$ — the user-specified frequency sweep
- $f_{i(actual)}(t)$ — the actual output frequency sweep, usually equal to $f_i(t)$
- $y(t)$ — the Chirp block output

$$y(t) = \cos(\psi(t) + \phi_0)$$

- $\psi(t)$ — the phase of the chirp signal, where $\psi(0) = 0$, and $2\pi f_i(t)$ is the derivative of the phase

$$f_i(t) = \frac{1}{2\pi} \cdot \frac{d\psi(t)}{dt}$$

Only holds for:

- Linear
- Quadratic
- Logarithmic

- ϕ_0 — the **Initial phase** parameter value, where $y_{chirp}(0) = \cos(\phi_0)$



Equations for Unidirectional Positive Sweeps:

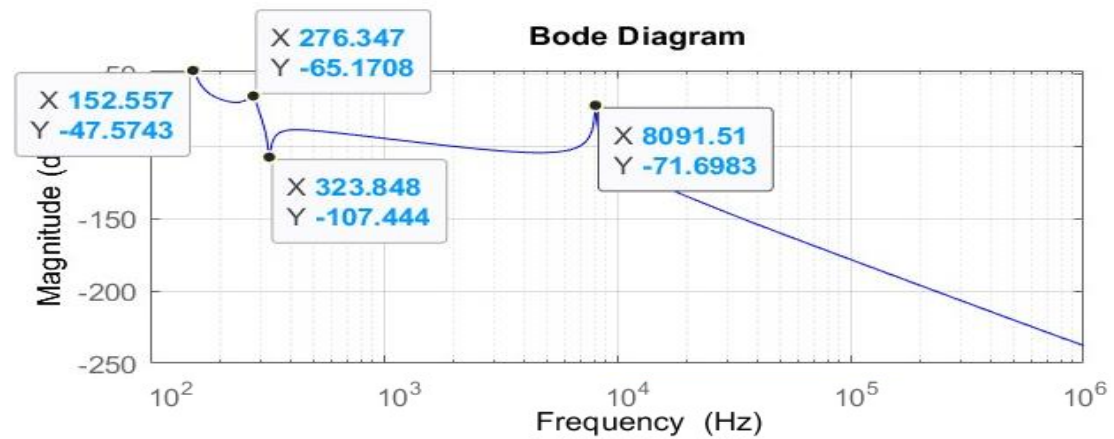
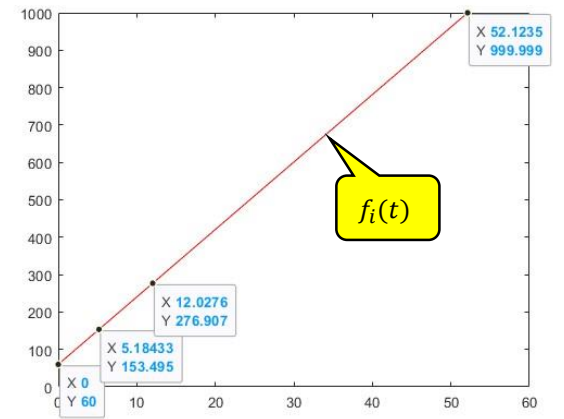
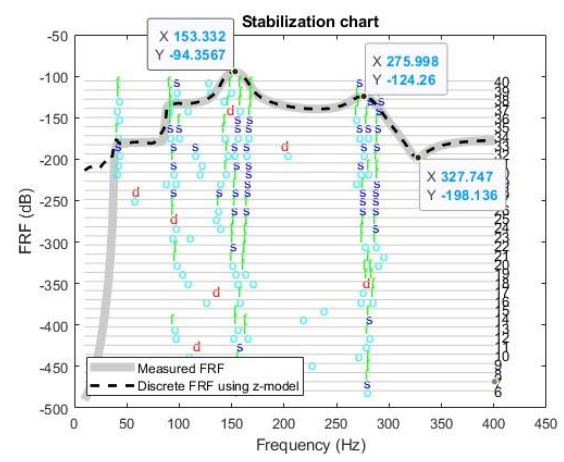
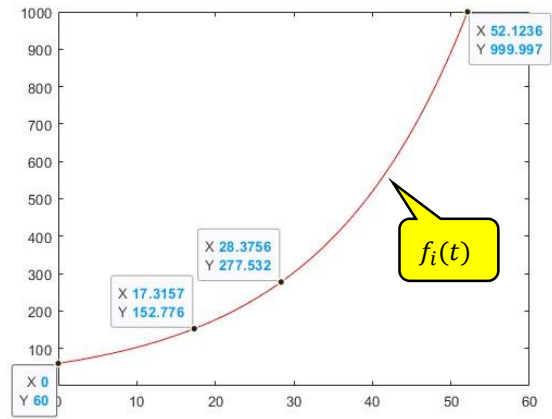
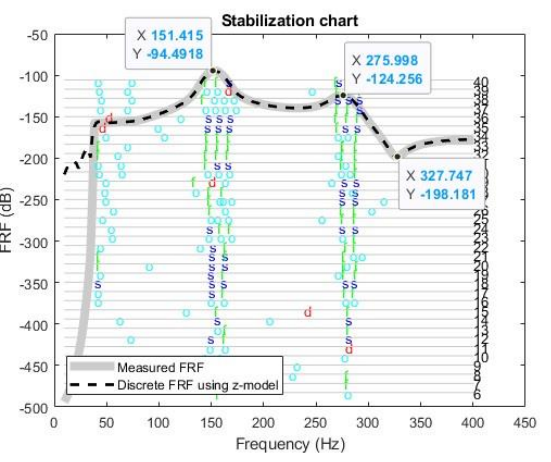
$$\rightarrow f_i(t) = \frac{1}{2\pi} \cdot \frac{d\psi(t)}{dt}$$

Frequency Sweep	Block Output Chirp Signal	User-Specified Frequency Sweep, $f_i(t)$	β	Actual Frequency Sweep, $f_{i(actual)}(t)$
Linear	$y(t) = \cos(\psi(t) + \phi_0)$	$f_i(t) = f_0 + \beta \cdot t$	$\beta = \frac{f_i(t_g) - f_0}{t_g}$	$f_{i(actual)}(t) = f_i(t)$
Quadratic	Same as Linear	$f_i(t) = f_0 + \beta t^2$	$\beta = \frac{f_i(t_g) - f_0}{t_g^2}$	$f_{i(actual)}(t) = f_i(t)$
Logarithmic	Same as Linear	$f_i(t) = f_0 \left(\frac{f_i(t_g)}{f_0}\right)^{\frac{t}{t_g}}$ Where $f_i(t_g) > f_0 > 0$	N/A	$f_{i(actual)}(t) = f_i(t)$
Swept cosine	$y(t) = \cos(2\pi f_i(t)t + \phi_0)$	Same as Linear	Same as Linear	$f_{i(actual)}(t) = f_i(t) + \beta t$



➤ **Logarithmic Instantaneous Frequency Sweep Rate:**

➤ **Linear Instantaneous Frequency Sweep Rate:**

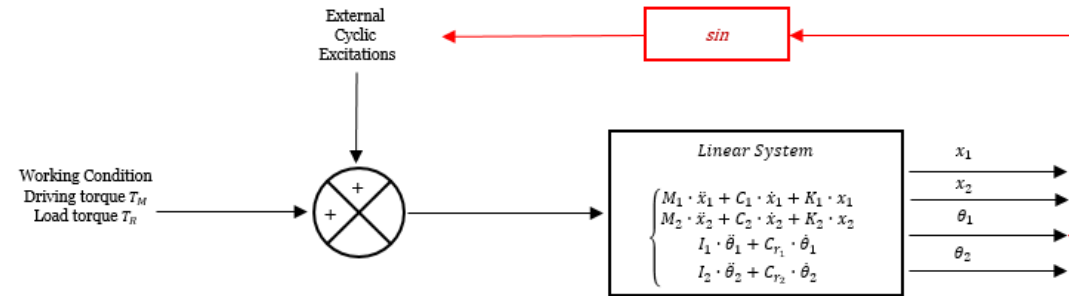




➤ A. Using “Cyclic external perturbation” + Ramp for excitation :

- Constant Gear Mesh Stiffness ($K_g = 1.8 \cdot e^8$)
- **Without** cyclic fluctuation ($\beta = 0$)

$$\begin{cases} M_1 \cdot \ddot{x}_1 + C_1 \cdot \dot{x}_1 + K_1 \cdot x_1 = -F_g \\ M_2 \cdot \ddot{x}_2 + C_2 \cdot \dot{x}_2 + K_2 \cdot x_2 = F_g \\ I_1 \cdot \ddot{\theta}_1 + C_{r1} \cdot \dot{\theta}_1 = \|T_M\| - R_{b1} \cdot F_g + T_{ex1} \\ I_2 \cdot \ddot{\theta}_2 + C_{r2} \cdot \dot{\theta}_2 = \|T_R\| - R_{b2} \cdot F_g \end{cases}$$



To have cyclic excitation, depending on the angular position of the shaft:

$$T_{ex1} = C_S \cdot \sin\left(f_{ev/tr} \cdot \theta_1(t)\right) + C_R \cdot t$$

Once we can use analogy and write ($\phi_0 = 0$):

$$y(t) = \cos\left(f_{ev/tr} \cdot \theta_1(t)\right)$$

Then the Instantaneous Frequency $f_i(t)$ becomes:

$$f_i(t) = \frac{f_{ev/tr}}{2\pi} \cdot \frac{d\theta_1(t)}{dt} = \frac{f_{ev/tr}}{2\pi} \cdot \dot{\theta}_1(t)$$

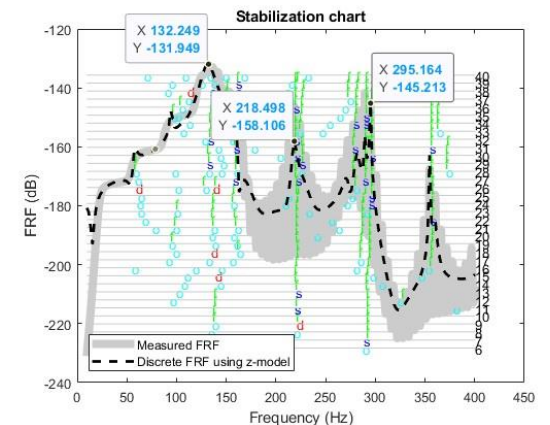
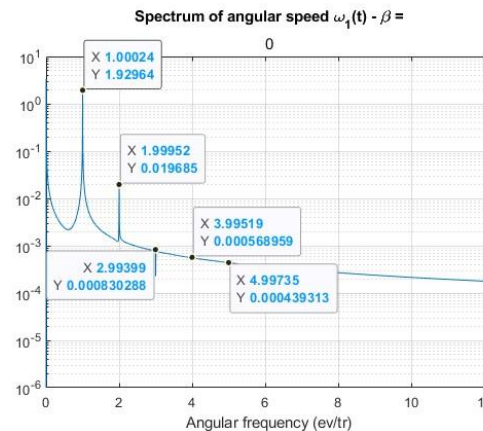
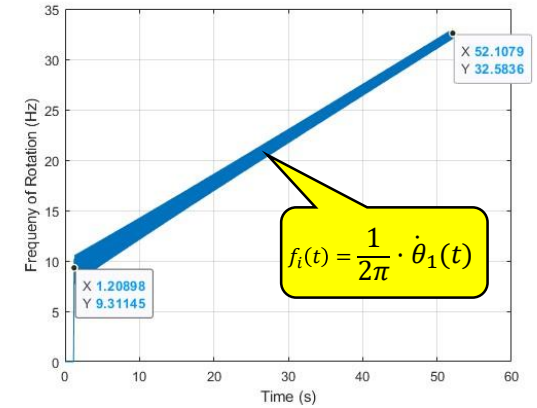
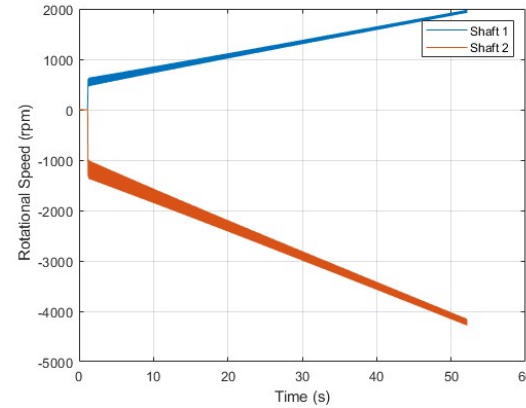
Case A.1:

- $T_{ex1} = C_S \cdot \sin(f_{ev/tr} \cdot \theta_1(t)) + C_R \cdot t$
- $C_S = 50$ & $C_R = 10$ & $f_{ev/tr} = 1$

According to the plot of the Frequency of the Rotation:

$$\left\{ \begin{array}{l} \bullet f_0 = 9.31 \text{ hz} \\ \bullet t_g = 52.1235 \text{ s} \\ \bullet f_i(t_g) = 32.58 \text{ hz} \end{array} \right.$$

- Since the frequency bandwidth covered by the Instantaneous frequency is not relevant to bandwidth of interest, the FRF estimation is not good!
- To cover a correct frequency bandwidth, the slope of ramp (C_R) needs to increase.



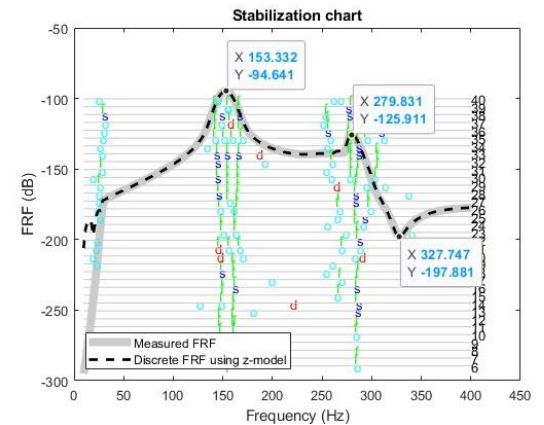
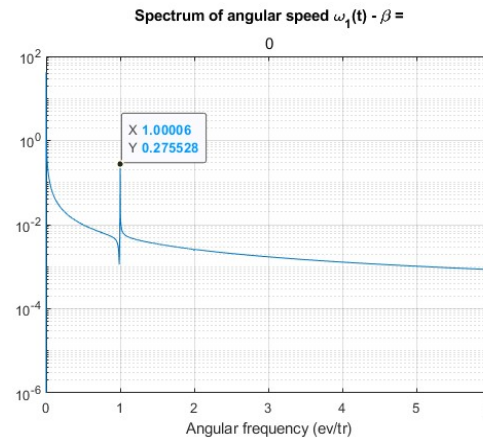
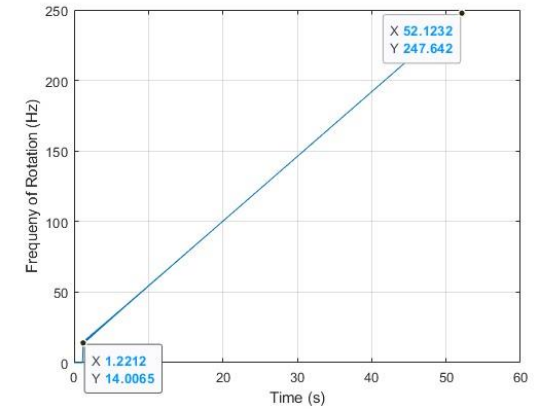
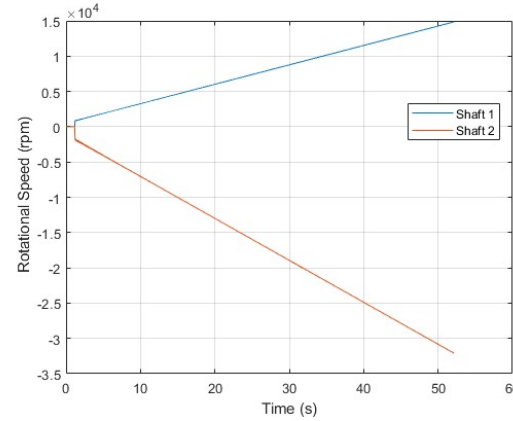
■ Case A.2:

- $T_{ex1} = C_S \cdot \sin(f_{ev/tr} \cdot \theta_1(t)) + C_R \cdot t$
- $C_S = 50$ & $C_R = 100$ & $f_{ev/tr} = 1$

According to the plot of the Frequency of the Rotation:

- $$\left\{ \begin{array}{l} \bullet f_0 = 14.006 \text{ hz} \\ \bullet t_g = 52.1235 \text{ s} \\ \bullet f_i(t_g) = 247.642 \text{ hz} \end{array} \right.$$

- Although the maximum frequency covered by the Instantaneous frequency is lower than the second resonance, but due to the transient response the FRF estimation is good enough!

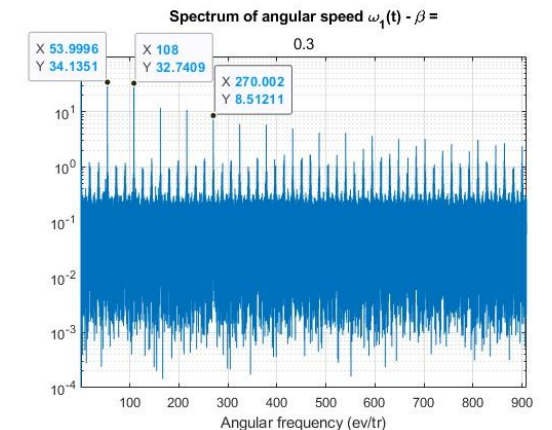
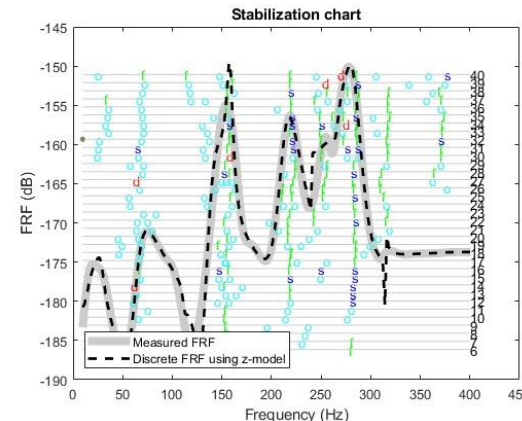
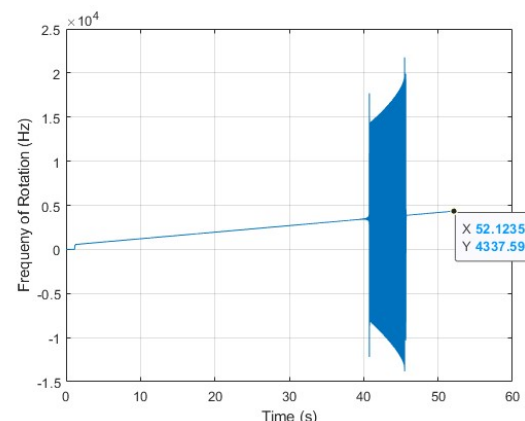
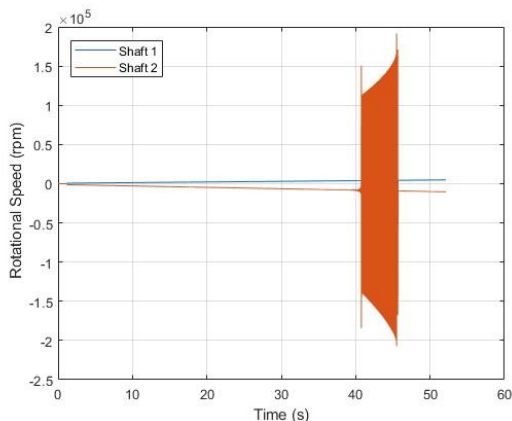
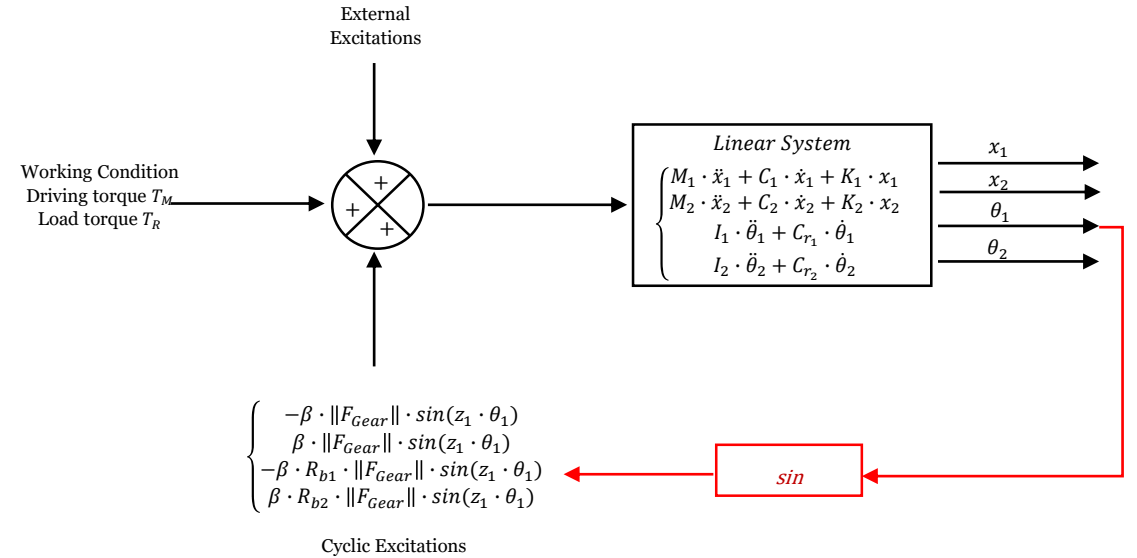


➤ B. Non-linear gear interaction:

$$\begin{cases} M_1 \cdot \ddot{x}_1 + C_1 \cdot \dot{x}_1 + K_1 \cdot x_1 = -F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \\ M_2 \cdot \ddot{x}_2 + C_2 \cdot \dot{x}_2 + K_2 \cdot x_2 = F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \\ I_1 \cdot \ddot{\theta}_1 + C_{r1} \cdot \dot{\theta}_1 = \|T_M\| - R_{b1} \cdot F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) + T_{ex1} \\ I_2 \cdot \ddot{\theta}_2 + C_{r2} \cdot \dot{\theta}_2 = \|T_R\| - R_{b2} \cdot F_g \cdot (1 + \beta \cdot \sin(z_1 \cdot \theta_1)) \end{cases}$$

- Varying Gear Mesh Stiffness (K_g)
- **With** cyclic fluctuation ($\beta = 0.3$)
- The external excitation (T_{ex1}) was chosen as: $T_{ex1} = C_R \cdot t$

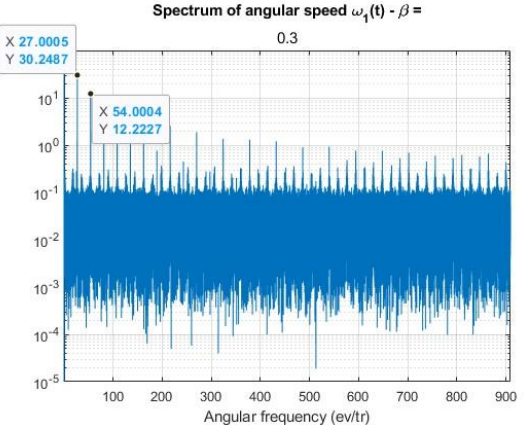
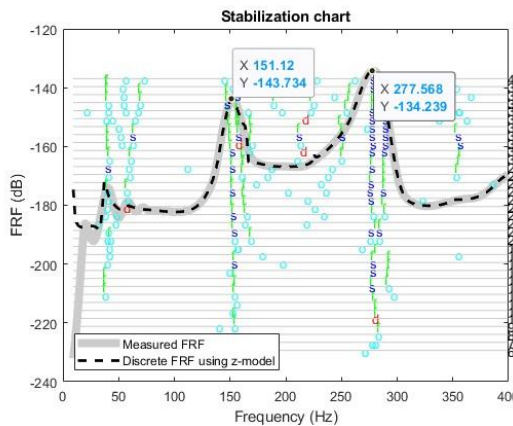
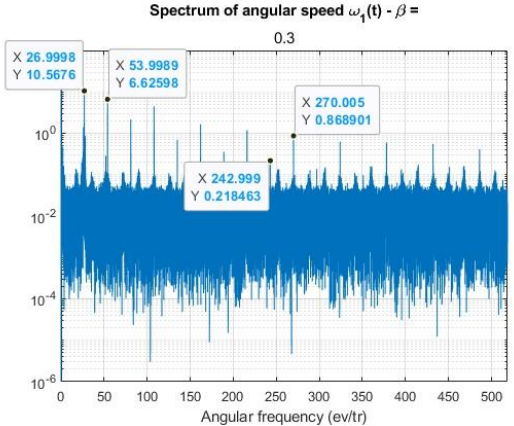
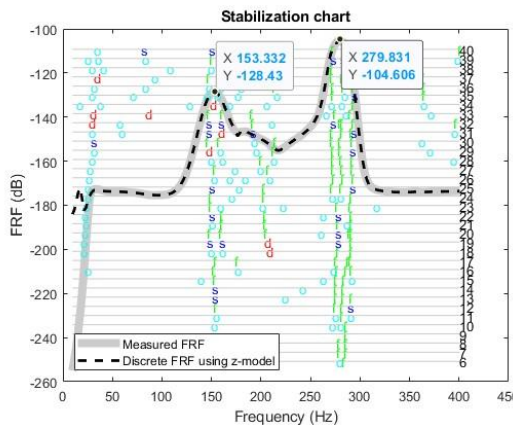
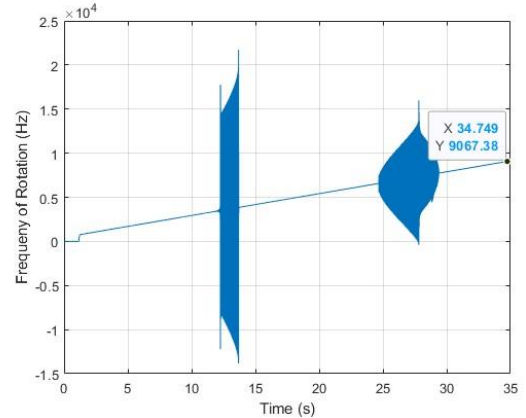
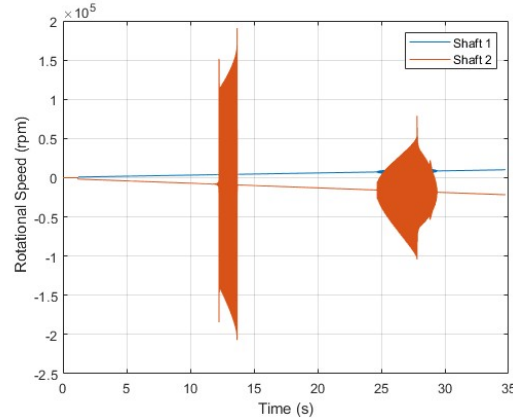
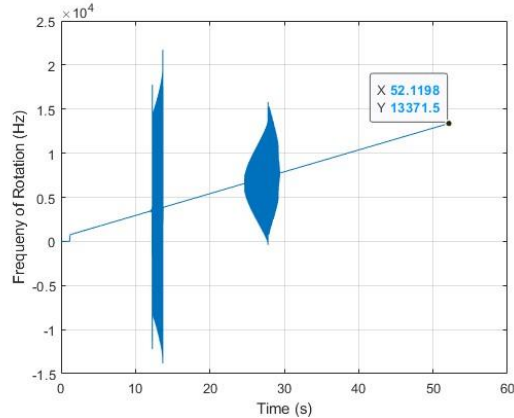
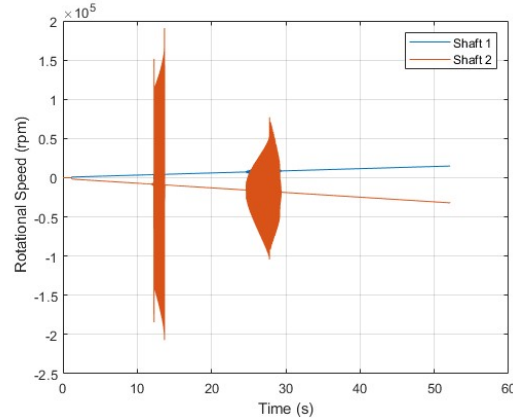
i. $\begin{cases} \bullet T_M = 1 \cdot 266 \text{ N} \cdot \text{m} \\ \bullet C_R = 30 \\ \bullet 450 \text{ rev} \end{cases}$





- ii. $\left\{ \begin{array}{l} \bullet T_M = 1 \cdot 266 \text{ N} \cdot \text{m} \\ \bullet C_R = 100 \\ \bullet 450 \text{ rev} \end{array} \right.$

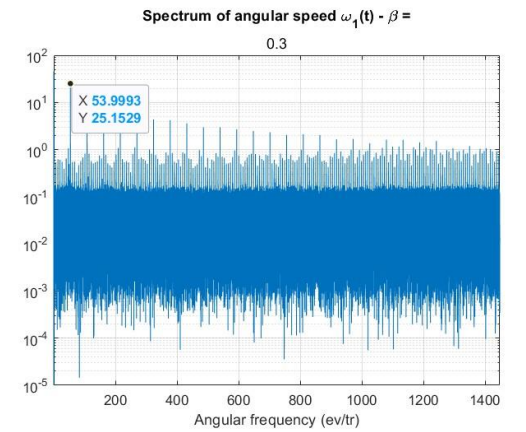
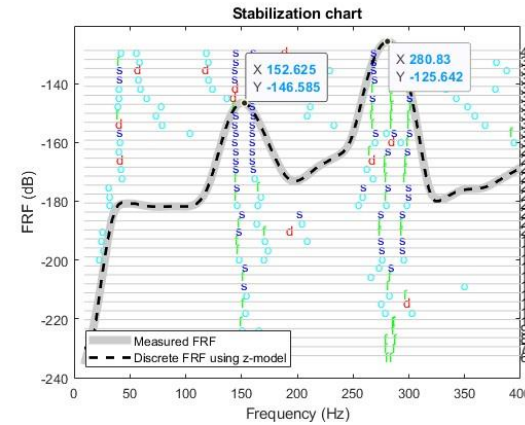
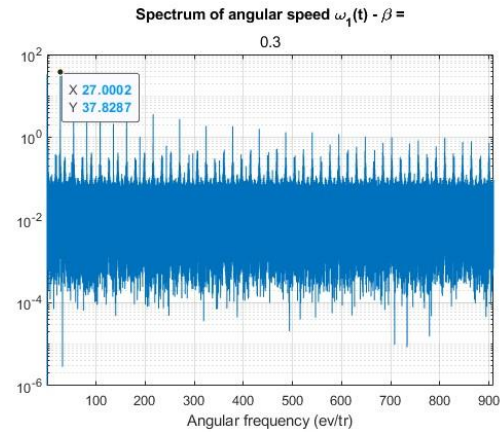
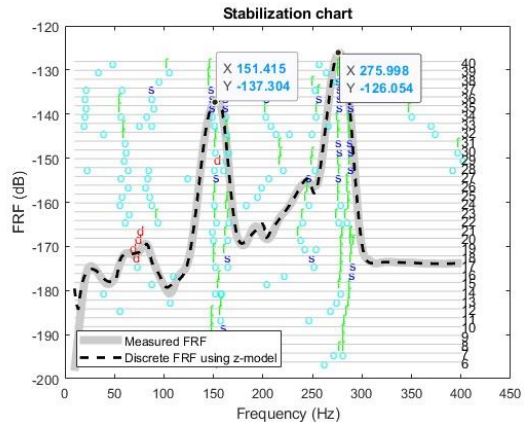
- iii. $\left\{ \begin{array}{l} \bullet T_M = 1 \cdot 266 \text{ N} \cdot \text{m} \\ \bullet C_R = 100 \\ \bullet 300 \text{ rev} \end{array} \right.$





$$iv. \begin{cases} \bullet T_M = 0.25 \cdot 266 \text{ N} \cdot \text{m} \\ \bullet C_R = 30 \\ \bullet 900 \text{ rev} \end{cases}$$

$$v. \begin{cases} \bullet T_M = 0.25 \cdot 266 \text{ N} \cdot \text{m} \\ \bullet C_R = 30 \\ \bullet 600 \text{ rev} \end{cases}$$





➤ C. Both the Non-linear gear interaction with external cyclic perturbation:

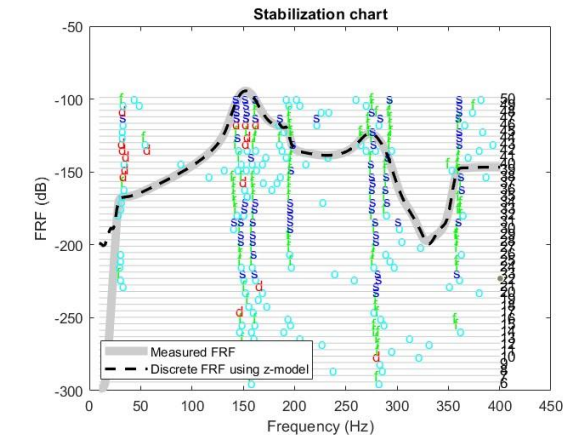
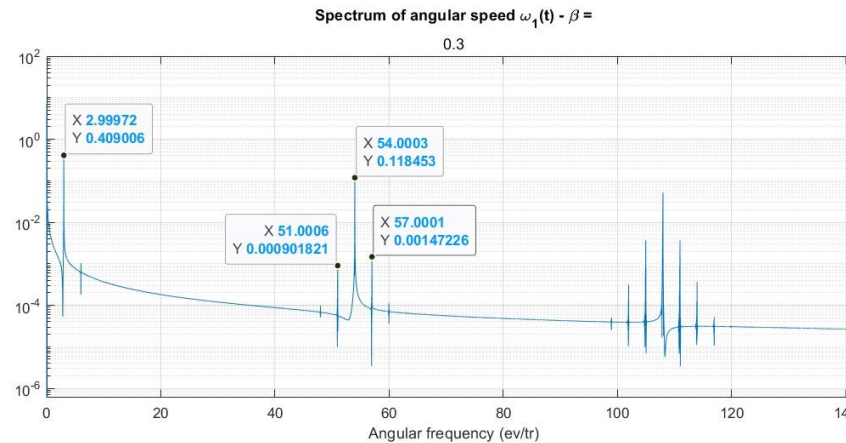
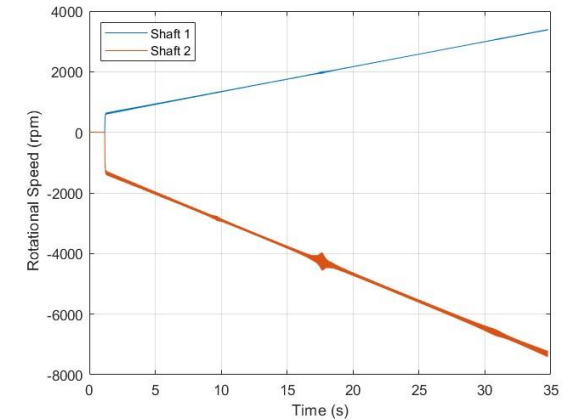
- Varying Gear Mesh Stiffness (K_g)
- **With** cyclic fluctuation ($\beta = 0.3$)
- External excitations are a cyclic perturbation, and a ramp in torque!

▪ Case C.1:

$$T_{ex1} = C_S \cdot \sin\left(f_{ev/tr} \cdot \theta_1(t)\right) + C_R \cdot t, \quad C_S = 50 \quad \& \quad C_R = 30$$

The Driving Torque: $T_M = 1 \cdot 266 \text{ N} \cdot \text{m}$

cyclic frequency $f_{ev/rev} = 3$



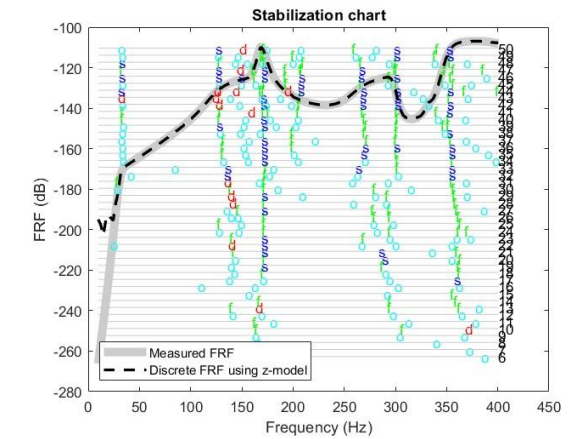
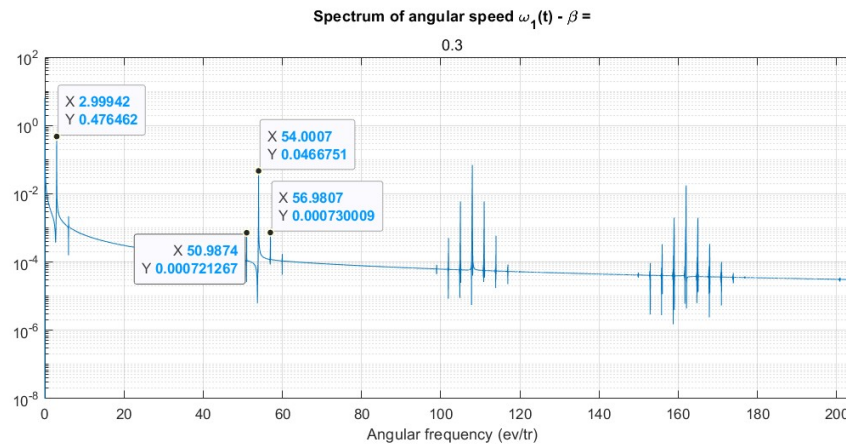
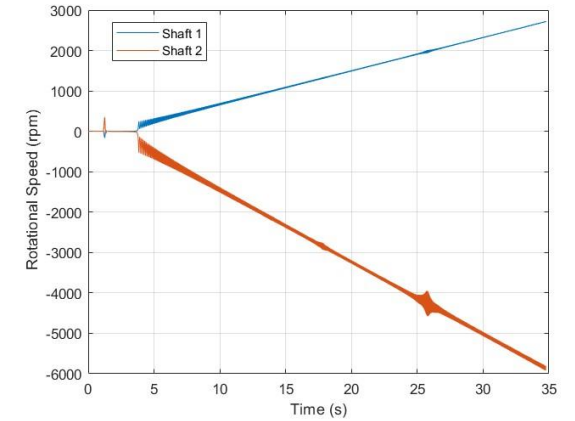
➤ C. Both the Non-linear gear interaction with external cyclic perturbation:

▪ Case C.2:

$$T_{ex1} = C_S \cdot \sin\left(f_{ev/tr} \cdot \theta_1(t)\right) + C_R \cdot t, \quad C_S = 50 \quad \& \quad C_R = 30$$

The Driving Torque: $T_M = 0.25 \cdot 266 \text{ N} \cdot \text{m}$

cyclic frequency $f_{ev/rev} = 3$



Questions:

In the presence of large frequency values, why half the fundamental harmonic appears? might be due to:

- Sampling: Not only coarse sampling induces error (Aliasing error), but also very fine sampling might cause difficulties too – what is the criterion for the upper limit of frequency sampling?
- Non-linear interaction inside the system? How to avoid it?



Thank You
For Your Attention

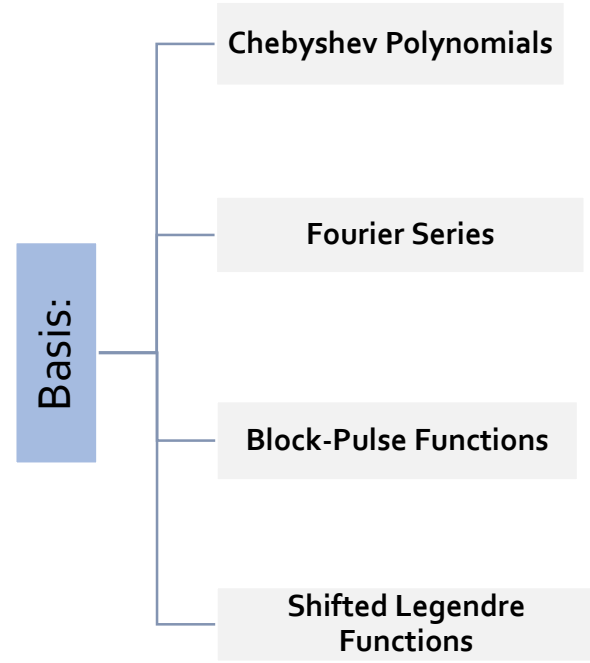


Introduction to the identification approaches

Time – Domain Orthogonal Functions

- Expanding of excitation and response signals in the polynomial basis
- Using integration and derivation property of the orthogonal functions
- so-called operational matrix of respectively integration and derivation

Differential equations can be transformed into algebraic equations



Book:

“Modelling and Identification with Rational Orthogonal Basis Functions” Springer, London. 2005

